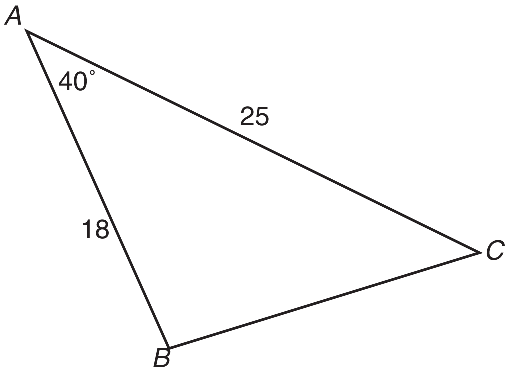
Unit 4: Geometry Lesson 4.5

Coach's Corner – VII

1. Determine the unknown side and the two unknown angles in the triangle shown.

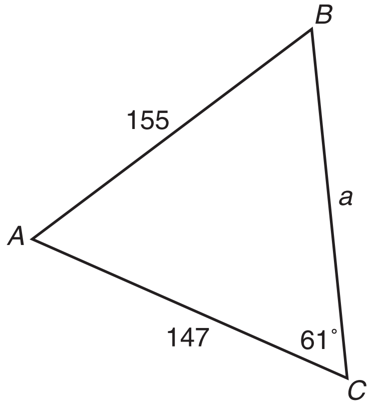


$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = 25^2 + 18^2 - 2(25)(18)\cos 40°$$
$$a^2 = 259.56\ldots$$
$$a = 16.11\ldots$$
$$a \doteq 16.1$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 40°}{16.11\ldots} = \frac{\sin B}{25}$$
$$\frac{25\sin 40°}{16.11\ldots} = \sin B$$
$$0.9974\ldots = \sin B$$
$$\sin^{-1} 0.9974\ldots = B$$
$$85.90\ldots° = B$$
$$85.9° \doteq B$$

$$A + B + C = 180°$$
$$40° + 85.90\ldots° + C = 180°$$
$$C = 54.09\ldots°$$
$$C \doteq 54.1°$$

2. In *Game On! 4.4*, you devised a strategy to solve for a in the triangle shown. In the process, you likely discovered that using the sine law to solve for a was fairly involved. Unfortunately, using the cosine law to solve this problem is also challenging.



a. Explain why it is difficult to use the cosine law to solve for a without determining any further information.

If you enter the information into the cosine law $c^2 = a^2 + b^2 - 2ab\cos C$, you get $155^2 = a^2 + 147^2 - 2a(147)\cos 61^\circ$. This equation is difficult to solve because it includes both a and a^2.

b. Rashid solved this problem using the cosine law as shown below. Explain the steps in his solution.

Step	Explanation
$c^2 = a^2 + b^2 - 2ab\cos C$	The angle C is given, so this version of the cosine law will be useful.
$155^2 = a^2 + 147^2 - 2a(147)\cos 61^\circ$	The values were substituted into the equation.
$24025 = a^2 + 21609 - 142.53\ldots a$	The terms have been simplified.
$0 = a^2 - 142.53\ldots a - 2416$	This is a quadratic equation. It has been rearranged from the previous step so one side is equal to zero. In this format, the quadratic formula can be used.
$a = \frac{142.53\ldots \pm \sqrt{(142.53\ldots)^2 - 4(1)(-2416)}}{2(1)}$	Values are substituted into the quadratic formula.
$a = \frac{142.53\ldots \pm \sqrt{29979.9\ldots}}{2}$	The equation is simplified.
$a = \frac{142.53\ldots \pm 173.14\ldots}{2}$	The equation is simplified further.
$a \doteq 157.8, \cancel{-15.3}$	The equation produces two solutions. The length cannot be negative, so the negative solution is ignored.
$a \doteq 157.8$	The approximate value of a is shown.

3. Having two eyes is important for depth perception. One reason is that your eyes need to look more inward when looking at an object nearby than they do for an object far away. One of the clues your brain uses to determine the distance to an object is the convergence angle between the line of sight from each eye.

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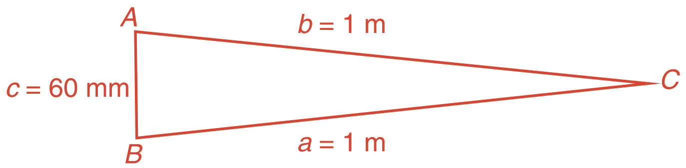
a. Jane has a pupillary distance of 60 mm. This means the pupils of her eyes are 60 mm apart. Determine the convergence angles for objects that are 1 m, 2 m, 100 m, and 200 m from her eyes (assume each eye is the same distance from the objects). Start by drawing a diagram for an object that is 1 m away. In this diagram, *A* and *B* represent the eyes and *C* represents the object the eyes converge on.



$\angle C$ can be determined using the cosine law. You will need to use the same unit for *a*, *b*, and *c*, so let 60 mm be expressed as 0.06 m.

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab\cos C \\ 0.06^2 &= 1^2 + 1^2 - 2(1)(1)(\cos C) \\ 0.0036 &= 2 - 2\cos C \\ 2\cos C &= 1.9964 \\ \cos C &= 0.9982 \\ C &= \cos^{-1} 0.9982 \\ C &\doteq 3.44° \end{aligned}$$

The same calculation can be repeated when a and b are both equal to 2, then 100, and then 200.

Distance to Eyes	1 m	2 m	100 m	200 m
Convergence Angle	3.44°	1.72°	0.0344°	0.0172°

b. Based on the information determined in part a, do you expect it to be easier to distinguish the distance between objects 1 m and 2 m away, or between objects 100 m and 200 m away? Explain.

For objects that are 1 m away and 2 m away, the convergence angles differ by 3.44° – 1.72° = 1.72°. For objects that are 100 m away and 200 m away, the convergence angles differ by 0.0344° – 0.0172° = 0.0172°. Based on convergence angles, it should be much easier to distinguish the distance between objects 1 m and 2 m away because the change in convergence angles is much larger.

Please complete *Lesson 4.5 Game On!*, *Unit 4 Time Out*, *Final Review Assignment*, and *Check Point!* located in *Workbook 4B*.