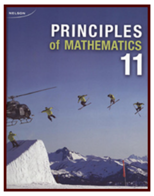
Lesson 4.5: The Cosine Law



Refer to *Principles of Mathematics 11* pages 150 and 162 for more examples.

- Pages 150, #1, 4a, 5a, 6a, 6c, 8, 10, 11a, and 12
- Pages 162, #6, 7, 9, 12, and 14

Question 1, p. 150

a. No, the cosine law requires two known sides and the contained angle to determine the third side. The sine law could be used to solve for *c*.

b. Yes, there is enough information to use the cosine law.

Question 4a, p. 151

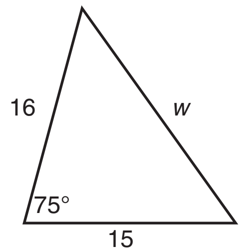
a.
$$b^2 = a^2 + b^2 - 2ab\cos B$$
$$b^2 = 9.5^2 + 10.5^2 - 2(9.5)(10.5)\cos 40^\circ$$
$$b^2 = 47.67\ldots$$
$$b = 6.90\ldots$$
$$b \doteq 6.9 \text{ cm}$$

Question 5a, p. 151

a.
$$p^2 = q^2 + r^2 - 2qr\cos P$$
$$2.2^2 = 3.9^2 + 3.5^2 - 2(3.9)(3.5)\cos\theta$$
$$-22.62 = -27.3\cos\theta$$
$$0.82\ldots = \cos\theta$$
$$\cos^{-1}0.82\ldots = \theta$$
$$34^\circ \doteq \theta$$

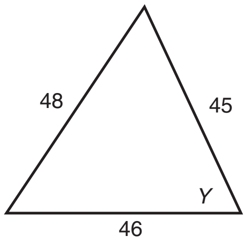
Question 6a and c, p. 151

a.



$$w^2 = 15^2 + 16^2 - 2(15)(16)\cos 75°$$
$$w^2 = 356.76\ldots$$
$$w = 18.88\ldots$$
$$w \doteq 18.9$$

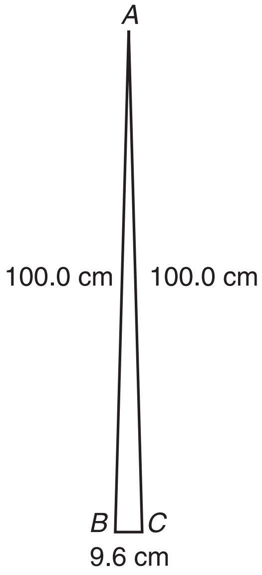
c.



$$48^2 = 46^2 + 45^2 - 2(46)(45)\cos Y$$
$$-1837 = -4140\cos Y$$
$$0.44\ldots = \cos Y$$
$$\cos^{-1} 0.44\ldots = Y$$
$$63.65\ldots° = Y$$
$$63.7° \doteq Y$$

Question 8, p. 152

a.



b.

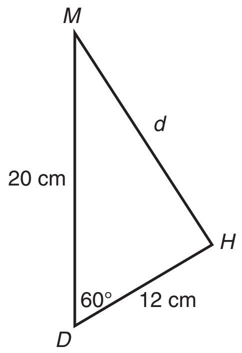
$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \\ 9.6^2 &= 100^2 + 100^2 - 2(100)(100)\cos A \\ -19907.84 &= -20000\cos A \\ 0.995392 &= \cos A \\ \cos^{-1}0.995392 &= A \\ 5.50\ldots^\circ &= A \\ 5.5^\circ \text{ cm} &\doteq A \end{aligned}$$

Question 10, p. 152

You can use the cosine law to solve this problem. Drawing the shorter diagonal produces a triangle where the diagonal is across the 70° angle and the other two sides are known.

Question 11a, p. 152

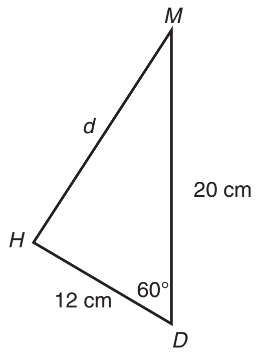
a. i.



At 2:00, the minute hand is at the 12 and the hour hand is $\frac{2}{12}$ of the way around the clock or $\frac{2}{12} \times 360^\circ = 60^\circ$ from the 12.

$$d^2 = 20^2 + 12^2 - 2(20)(12)\cos 60^\circ$$
$$d^2 = 304$$
$$d = 17.43\ldots$$
$$d \doteq 17.4 \text{ cm}$$

ii. At 10:00, the minute hand is at the 12 and the hour hand is also $\frac{2}{12}$ of the way around the clock, or 60°, when going in a counter-clockwise direction.



$$d^2 = 20^2 + 12^2 - 2(20)(12)\cos 60^\circ$$
$$d^2 = 304$$
$$d = 17.43\ldots$$
$$d \doteq 17.4 \text{ cm}$$

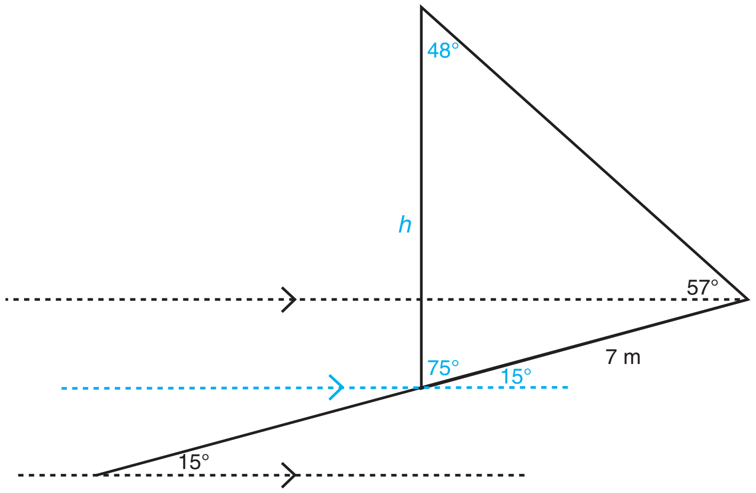
Question 12, p. 152

No, it is impossible to make a triangle if one side is longer than the sum of the lengths of the other two sides.

Question 6, p. 162

a. Drawing a horizontal line at the base of the tree shows corresponding angles of 15˚. This means the angle formed by the vertical tree and the ground is 75˚ and the third angle of the triangle is 15˚ using the sum of the interior angles in a triangle. This gives you enough information to use the sine law to determine the height of the tree.

b.



$$\frac{h}{\sin 57^\circ} = \frac{7}{\sin 48^\circ}$$

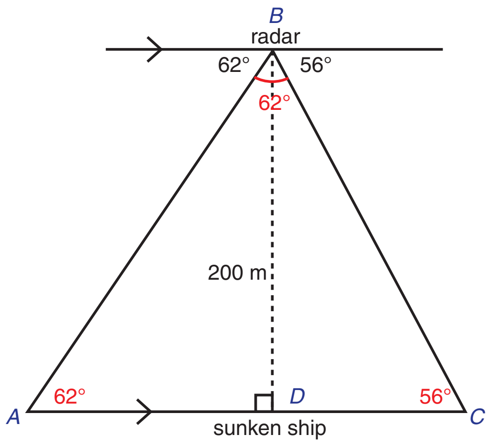
$$h = \frac{7\sin 57^\circ}{\sin 48^\circ}$$

$$h = 7.89\ldots$$

$$h = 8 \text{ m}$$

Question 7, p. 162

Using alternate interior angles and the sum of the interior angles in a triangle, the three angles in the triangle can be found as shown in the diagram.

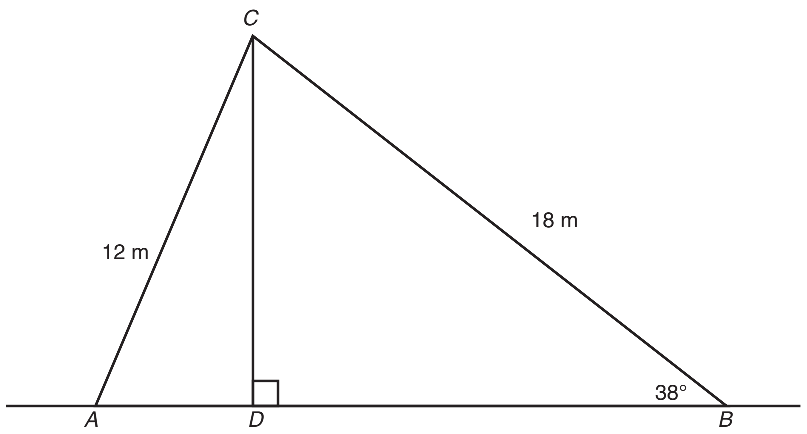


$$\sin A = \frac{BD}{AB}$$
$$\sin 62^\circ = \frac{200}{AB}$$
$$AB = \frac{200}{\sin 62^\circ}$$
$$AB = 226.51\ldots$$

$$\frac{AC}{\sin \angle ABC} = \frac{AB}{\sin \angle C}$$
$$\frac{AC}{\sin 62^\circ} = \frac{226.51\ldots}{\sin 56^\circ}$$
$$AC = \frac{(226.51\ldots)\sin 62^\circ}{\sin 56^\circ}$$
$$AC = 241.24\ldots$$
$$AC \doteq 241.2 \text{ m}$$

Question 9, p. 162

a.



$$\sin B = \frac{CD}{BC}$$
$$\sin 38^\circ = \frac{CD}{18}$$
$$18 \sin 38^\circ = CD$$
$$11.08\ldots = CD$$
$$11.1 \text{ m} \doteq CD$$

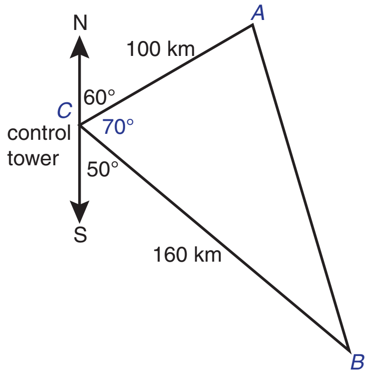
b.
$$AD^2 + DC^2 = AC^2$$
$$AD^2 + 11.08\ldots^2 = 12^2$$
$$AD^2 = 21.19\ldots$$
$$AD = 4.60\ldots$$

$$\cos B = \frac{BD}{BC}$$
$$\cos 38^\circ = \frac{BD}{18}$$
$$18\cos 38^\circ = BD$$
$$14.18\ldots = BD$$

$$AD + BD = AB$$
$$4.60\ldots + 14.18\ldots = AB$$
$$18.78\ldots = AB$$
$$18.8 \text{ m} \doteq AB$$

Question 12, p. 163

a.



$$AB^2 = AC^2 + BC^2 - 2(AC)(BC)\cos\angle ACB$$
$$AB^2 = 100^2 + 160^2 - 2(100)(160)\cos 70^\circ$$
$$AB^2 = 24655.35\ldots$$
$$AB = 157.02\ldots$$
$$AB \doteq 157.0 \text{ km}$$

b. The plane that is 100 km away will arrive first.

Question 14, p. 163

Strategies may vary, a sample is described.

- Determine $\angle BDC$ using the sum of the interior angles in a triangle.
- Determine the length of CD using the sine law.
- Determine the height of the blimp using the tangent ratio in right triangle ACD.

$$\begin{aligned}\angle CBD + \angle BCD + \angle BDC &= 180^\circ \\ 50^\circ + 66^\circ + \angle BDC &= 180^\circ \\ \angle BDC &= 64^\circ\end{aligned}$$

$$\begin{aligned}\frac{CD}{\sin \angle DBC} &= \frac{BC}{\sin \angle BDC} \\ \frac{CD}{\sin 50^\circ} &= \frac{175.0}{\sin 64^\circ} \\ CD &= \frac{175.0 \sin 50^\circ}{\sin 64^\circ} \\ CD &= 149.15\ldots\end{aligned}$$

$$\begin{aligned}\tan \angle ACD &= \frac{h}{CD} \\ \tan 74^\circ &= \frac{h}{149.15\ldots} \\ (149.15\ldots)\tan 74^\circ &= h \\ 520.15\ldots &= h \\ 520.2 \text{ m} &\doteq h\end{aligned}$$