Practice Run

1. Which is the better buy? Show all work.

   a. 12 cans of pop for $6.99 or
   32 cans of pop for $14.99

   b. 4.4 pounds of sugar for $4.99 or
   10 pounds of sugar for $9.25

2. Grant wants to make meatballs for his pasta supper. He needs to defrost 1.56 kg of ground beef. If it takes 12 minutes to defrost 2 lbs of meat, how long will it take for Grant's 1.56 kg of ground beef to defrost?
   1 kg = 2.2 lb

3. A professional cyclist races at an average speed of 40 km/h and burns 2500 calories per hour in a 4-hour race. If an amateur cyclist races at an average of 30 km/h and burns over 7000 calories in a 4-hour race, how many more minutes must the amateur ride at his race pace to burn the same number of calories as the professional in his 4-hour race?

Compare your answers.

1. Which is the better buy? Show all work.

a. 12 cans of pop for \$6.99 or
32 cans of pop for \$14.99

$$\frac{\$6.99}{12 \text{ cans}} = \$0.58/\text{can}$$

$$\frac{\$14.99}{32 \text{ cans}} = \$0.47/\text{can}$$

The better buy is the 32 cans of pop, since the unit price for 32 cans is \$0.47/can.

b. 4.4 pounds of sugar for \$4.99 or
10 pounds of sugar for \$9.25

$$\frac{\$4.99}{4.4 \text{ lbs}} = \$1.13/\text{lb}$$

$$\frac{\$9.25}{10 \text{ lbs}} = \$0.93/\text{lb}$$

The better buy is the 10 pounds of sugar, since the unit price for 10 pounds is \$0.93/pound.

2. Grant wants to make meatballs for his pasta supper. He needs to defrost 1.56 kg of ground beef. If it takes 12 minutes to defrost 2 lbs of meat, how long will it take for Grant's 1.56 kg of ground beef to defrost?
1 kg = 2.2 lb

Convert kg to pounds

$$2.2 \text{ lbs} = 1 \text{ kg}$$

$$x \text{ lb} = 1.56 \text{ kg}$$

$$\frac{x}{1.56 \text{ kg}} = \frac{2.2 \text{ lbs}}{1 \text{ kg}}$$

$$x = \frac{2.2 \text{ lbs} \bullet 1.56 \text{ kg}}{1 \text{ kg}}$$

$$x = 3.432 \text{ lbs}$$

Find the time to defrost 3.432 lb of ground beef.

$$2 \text{ lbs}/12 \text{ minutes}$$

$$3.432 \text{ lbs}/t \text{ minutes}$$

$$\frac{2 \text{ lbs}}{12 \text{ minutes}} = \frac{3.432 \text{ lbs}}{t \text{ minutes}}$$

$$2 \text{ lbs} \bullet t = 12 \text{ minutes} \bullet 3.432 \text{ lbs}$$

$$\frac{2 \text{ lbs}}{2 \text{ lbs}} t = \frac{12 \text{ minutes} \bullet 3.432 \text{ lbs}}{2 \text{ lbs}}$$

$$t = 20.592 \text{ minutes}$$

It will take 20.6 minutes to defrost 3.432 lbs of ground beef.

3. A professional cyclist races at an average speed of 40 km/h and burns 2500 calories per hour in a 4-hour race. If an amateur cyclist races at an average of 30 km/h and burns over 7000 calories in a 4-hour race, how many more minutes must the amateur ride at his race pace to burn the same number of calories as the professional in his 4-hour race?

Professional	~~40 km/h~~	2500 cal/h
Amateur	~~30 km/h~~	7000 cal/4h

The actual speeds of the cyclists do not factor into this particular problem.

Step 1: Find the unit rate (number of calories per hour) for the amateur cyclist burning 7000 calories in 4 hours.

$$\frac{7000\text{ cal}}{4\text{ h}} = \frac{1750\text{ cal}}{1\text{ h}}$$

Step 2: Determine the difference between the numbers of calories burned by each athlete in one hour.

2500 – 1750 = 750

The amateur burns 750 fewer calories per hour than the professional.
In four hours, the amateur will burn $750 \times 4 = 3000$ fewer calories.

Step 3: Determine how long it will take the amateur to burn 3000 calories.

$$\frac{1750\text{ cal}}{1\text{ h}} = \frac{3000\text{ cal}}{t}$$
$$1750\text{ cal} \cdot t = 3000\text{ cal} \cdot 1\text{ h}$$
$$\frac{1750\text{ cal} \cdot t}{1750\text{ cal}} = \frac{3000\text{ cal} \cdot 1\text{ h}}{1750\text{ cal}}$$
$$t \doteq 1.71\text{ h}$$

The amateur will need to ride an additional 1.71 hours to burn the same number of calories as the professional in his 4-hour race.

D. Interpreting the Slope of a Line as a Rate

Slope represents the relationship between the change in vertical values (rise) and the corresponding change in horizontal values (run). Slope is also commonly represented by m.

slope
Represents the relationship (as a ratio) between the change in vertical values (rise) and the corresponding change in horizontal values (run) on a graph
$m = \frac{\Delta y}{\Delta x} = \frac{\text{vertical change}}{\text{horizontal change}}$
$m = \frac{\text{rise}}{\text{run}}$
$m = \frac{y_2 - y_1}{x_2 - x_1}$

Slope can be represented in the following ways:

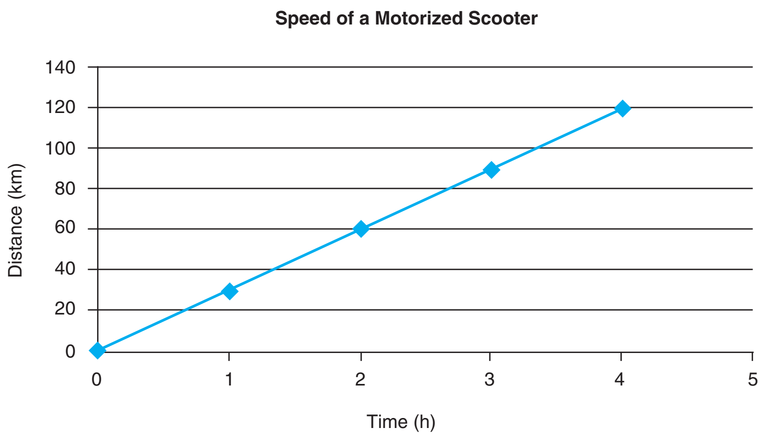
$$m = \frac{\Delta y}{\Delta x} = \frac{\text{vertical change}}{\text{horizontal change}}$$

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Because slope relates the independent and dependent variables, slope can also be described as a rate of change.

A distance vs. time graph shows the distance travelled in a certain time interval. Kilometres per hour (distance/time) is a rate called speed or velocity.



Example 1

Using the graph above, determine the unit rate for the speed of the motorized scooter.

The graph shows distance, in km, versus time, in hours. By picking two known points on the straight-line graph, you can determine the change in both time and distance and use those values to calculate the rate (speed).

One clearly marked point on the graph is the distance of 0 km when the time is 0 hours. Another clearly marked point on the graph is the distance of 60 km when the time is 2 hours.

continued…

Example 1

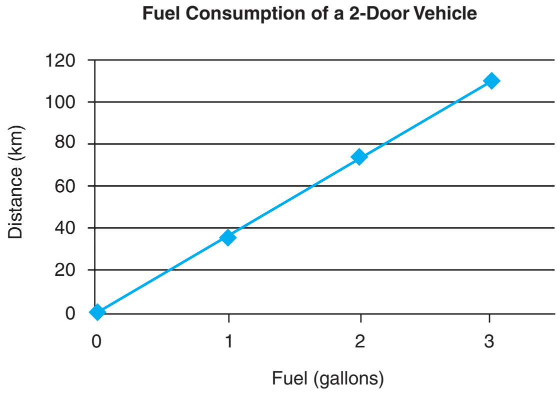
...continued

Therefore, a distance of 60 km – 0 km = 60 km was travelled in a time of 2 h – 0 h = 2 h.

$$m = \frac{60 - 0}{2 - 0}$$
$$m = \frac{60}{2}$$
$$m = \frac{30}{1}$$
$$m = \frac{30\text{ km}}{1\text{ hour}}$$

The scooter's rate of speed was 30 km/h.

The fuel consumption of a certain vehicle can be expressed as the number of miles travelled per gallon of liquid fuel.



Example 2

Given that a 2-door vehicle averages 37 miles per gallon in the graph above, how many miles could be driven using 4.5 gallons of gasoline?

Set up a proportion relating the average fuel consumption, as a unit rate, to the number of miles, n, travelled on 4.5 gallons.

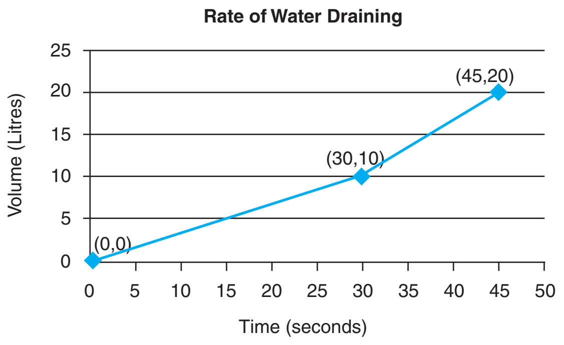
$$\frac{37}{1} = \frac{n}{4.5}$$
$$37 \bullet 4.5 = 1n$$
$$166.5 = n$$

A 2-door vehicle can travel 166.5 miles using 4.5 gallons of fuel.

Example 3

As a tank of water is drained, 10 L will flow out in the first 30 seconds and then another 10 L will flow out in the next 15 seconds, as illustrated in the graph below.

- Create a graph representing the situation described. Be sure to plot the amount of water drained versus time and label the graph.
- Determine the slope for each time interval – the first 30 seconds and then the next 15 seconds.
- Analyze what is happening during each interval.



Slope for the first 30 seconds.

$$m = \frac{\text{change in volume of water}}{\text{change in time}}$$

$$m = \frac{v_2 - v_1}{t_2 - t_1}$$

$$m = \frac{10 - 0}{30 - 0}$$

$$m = \frac{10}{30} = \frac{1}{3} = 0.3333\ldots$$

Slope for the first 30 seconds is 0.33 L/sec.

Slope for the next 15 seconds.

$$m = \frac{\text{change in volume of water}}{\text{change in time}}$$

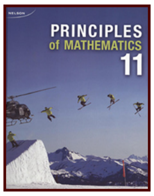
$$m = \frac{v_2 - v_1}{t_2 - t_1}$$

$$m = \frac{20 - 10}{45 - 30}$$

$$m = \frac{10}{15} = \frac{2}{3} = 0.6666\ldots$$

Slope for the next 15 seconds is 0.67 L/sec.

For the second time interval, between 30 s and 45 s, the unit rate is larger and the slope of the graph is steeper. As such, the water flowed out of the tank faster in the second interval.



For further information about problems involving rates see p. 454 to 458 of *Principles of Mathematics 11*.