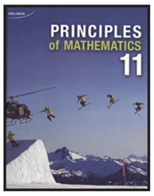
Lesson 5.2 Scale Factors and Scale Diagrams



Refer to *Principles of Mathematics 11* pages 467-470 and 484-488 for more examples.

- Page 471, #3, 4, 11, 13a, 14, 17, 18
- Page 489, #1, 2, 6, 8, 9, 11, and 13

Question 3, Page 471

a. The scale of a plan is represented by a ratio.
It can either be expressed as 5 inches:6 feet or where both measurements are in inches as follows:
1 foot = 12 inches

$$\frac{x}{6\text{ ft}} = \frac{12\text{ in}}{1\text{ ft}}$$

$$x = \frac{6\text{ ft} \cdot 12\text{ in}}{1\text{ ft}}$$

$$x = 72\text{ in}$$

5 : 72

b. $$k = \frac{\text{diagram measurement}}{\text{actual measurement}}$$

$$k = \frac{5\text{ in}}{72\text{ in}}$$

$$k = \frac{5}{72}$$

Question 4, Page 471

Convert one centimetre measure to metres to determine the scale factor.

$\frac{x}{6\text{ cm}} = \frac{1\text{ m}}{100\text{ cm}}$ $\frac{x}{6\text{ cm}} = \frac{6\text{ cm} \bullet 1\text{ m}}{100\text{ cm}}$ $x = 0.06\text{ m}$		scale factor $= \frac{\text{larger shape}}{\text{smaller shape}}$ $k = \frac{9\text{ m}}{0.06\text{ m}}$ $k = 150$	
Note: To determine the scale factor, convert values to the same units.			
$8.0\text{ m} = 800\text{ cm}$ $h = ?$ $\text{k} = \frac{\text{larger}}{\text{smaller}}$ $150 = \frac{800\text{ cm}}{h}$ $150h = 800\text{ cm}$ $\frac{150}{150}h = \frac{800}{150}$ $h = 5.\bar{3}\text{ cm}$ or $5\frac{1}{3}$ cm	$6.0\text{ m} = 600\text{ cm}$ $g = ?$ $k = \frac{\text{larger}}{\text{smaller}}$ $150 = \frac{600\text{ cm}}{g}$ $150g = 600\text{ cm}$ $\frac{150}{150}g = \frac{600\text{ cm}}{150}$ $g = 4.0\text{ cm}$	$4.0\text{ cm} = 0.04\text{ m}$ $x = ?$ $k = \frac{\text{larger}}{\text{smaller}}$ $150 = \frac{x}{0.04\text{ m}}$ $150 \bullet 0.04\text{ m} = x$ $6.0\text{ m} = x$	$5.0\text{ cm} = 0.05\text{ m}$ $y = ?$ $k = \frac{\text{larger}}{\text{smaller}}$ $150 = \frac{y}{0.05\text{ m}}$ $150 \bullet 0.05\text{ m} = y$ $7.5\text{ m} = y$

Question 11, Page 473

$$\text{scale factor} = \frac{\text{image measurement}}{\text{actual measurement}}$$

$$1\text{ cm} = 10\text{ mm}$$

$$40 = \frac{10\text{ mm}}{x}$$

$$40x = 10\text{ mm}$$

$$\frac{40}{40}x = \frac{10\text{ mm}}{40}$$

$$x = \frac{1}{4}\text{ mm} \quad \text{or} \quad 0.25\text{ mm}$$

Question 13a, Page 473

The scale is 1:75. Since units are not included, the scale can be applied to any unit of measure. Since the grid paper is measured in cm, use 1 cm = 75 cm.
If 1 square = 0.5 cm, then 2 squares = 1 cm.
Determine the perimeter of the diagram.

$$P = 6\text{ cm} + 4\text{ cm} + 3\text{ cm} + 1\text{ cm} + 3\text{ cm} + 3\text{ cm} = 20\text{ cm}$$

$$k = \frac{\text{diagram}}{\text{actual}}$$

Let x = actual perimeter

$$\frac{1}{75} = \frac{20\text{ cm}}{x}$$

$$x = 20\text{ cm} \cdot 75$$

$$x = 1500\text{ cm}$$

The perimeter of the greenhouse is 1500 cm or 15 metres.

Question 14, Page 474

a. 5.7 cm = 57 mm

$$k = \frac{\text{diagram}}{\text{actual}}$$

$$k = \frac{57\text{ mm}}{19\text{ mm}}$$

$$k = 3$$

b. $1\frac{1}{2}$ inches = 1.5 inches

$$k = \frac{\text{diagram}}{\text{actual}}$$

$$k = \frac{1.5\text{ in}}{30\text{ in}}$$

$$k = 0.05 \quad \text{or} \quad \frac{1}{20}$$

c. 2.5 cm = 0.025 m

$$k = \frac{\text{diagram}}{\text{actual}}$$

$$k = \frac{1}{0.025\text{ m}}$$

$$k = 40$$

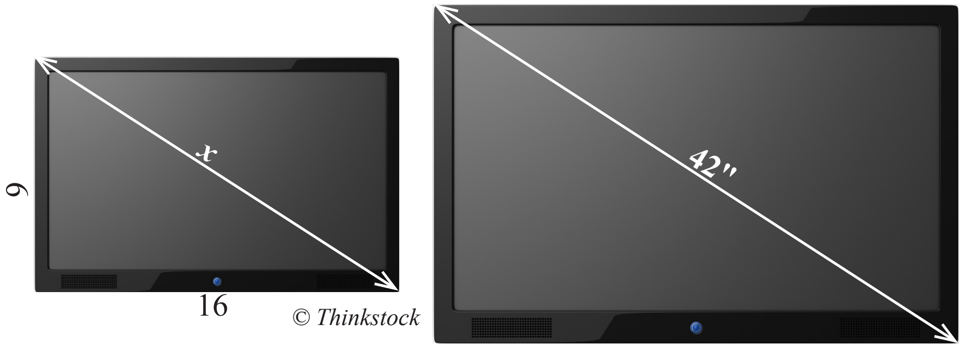
d. 55 feet = 660 inches (55×12)

$$k = \frac{\text{diagram}}{\text{actual}}$$

$$k = \frac{6 \text{ in}}{660 \text{ in}}$$

$$k = \frac{1}{110}$$

Question 17, Page 474



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Similar rectangles Using Pythgorean's theorem, find x.	Scale factor	height	length
$x = \sqrt{a^2 + b^2}$ $x = \sqrt{9^2 + 16^2}$ $x \doteq 18.4$ in	$k \doteq \frac{42 \text{ in}}{18.4 \text{ in}}$ $k \doteq 2.3$	$k = \frac{\text{enlargement}}{\text{actual}}$ $2.3 \doteq \frac{h}{9 \text{ in}}$ $2.3 \bullet 9 \text{ in} \doteq h$ $20.6 \text{ in} \doteq h$	$k = \frac{\text{enlargement}}{\text{actual}}$ $2.3 \doteq \frac{l}{16 \text{ in}}$ $2.3 \bullet 16 \text{ in} \doteq h$ $36.6 \text{ inches} \doteq h$

Width of bookshelves

12 inches = 1 foot

$$\frac{12 \text{ in}}{1 \text{ ft}} = \frac{x}{4 \text{ ft}}$$

$$x = \frac{12 \text{ in} \times 4 \text{ ft}}{1 \text{ ft}}$$

$$x = 48 \text{ in}$$

The book shelves are 26 inches in height and 48 inches wide. The 42 inch TV is 20.6 inches in height and 36.6 inches in length, so the TV will fit.

Question 18, Page 474

a. Any shaped logo that is contained in a 12 cm by 12 cm space.

b.

$k = \frac{25}{100}$ $k = \frac{1}{4}$	Determine the dimensions of the 12 cm × 12 cm logo in the scale diagram. $k = \frac{\text{diagram}}{\text{actual}}$ $\frac{1}{4} = \frac{d}{12 \text{ cm}}$ $\frac{12 \text{ cm}}{4} = d$ $3 \text{ cm} = d$

The scale diagram here should be similar to the logo from part a., but contained in a 3 cm by 3 cm space.

c.

$k = 1.5$	Determine the dimensions of the 12 cm × 12 cm logo in the scale diagram. $k = \frac{\text{diagram}}{\text{actual}}$ $1.5 = \frac{d}{12 \text{ cm}}$ $1.5 \cdot 12 \text{ cm} = d$ $18 \text{ cm} = d$

The scale diagram here should be similar to the logo from part a., but contained in a 18 cm by 18 cm space.

Question 1, Page 489

a. The two objects are similar. Each of the three dimensions is enlarged by the same scale factor.

$k = \frac{3 \text{ cm}}{2 \text{ cm}}$

$k = 1.5$

b. The two objects are similar. Each of the corresponding sides is reduced by the same scale factor.

$k = \frac{2\text{ cm}}{4\text{ cm}}$ $k = \frac{0.5\text{ cm}}{1\text{ cm}}$ $k = \frac{1\text{ cm}}{2\text{ cm}}$

$k = 0.5$ $k = 0.5$ $k = 0.5$

c. The two objects are similar. Both the corresponding heights and radii are reduced by the same scale factor.

$k = \frac{3\text{ cm}}{9\text{ cm}}$ $k = \frac{2\text{ cm}}{6\text{ cm}}$

$k = \frac{1}{3}$ $k = \frac{1}{3}$

d. The two objects are not similar. The corresponding dimensions are not reduced by the same scale factor.

$k = \frac{2\text{ cm}}{5\text{ cm}}$ $k = \frac{5\text{ cm}}{10\text{ cm}}$

$k = \frac{2}{5}$ $k = \frac{1}{2}$ $\frac{2}{5} \neq \frac{1}{2}$

Question 2, Page 490

a. All spheres are similar because the radius is the only measure that affects the size of a sphere.

b.

i) NBA ball:WNBA ball	ii) WNBA ball:NBA ball
$k = \frac{25\text{ cm}}{22\text{ cm}}$	$k = \frac{22\text{ cm}}{25\text{ cm}}$
$k = 1.1\overline{36}$	$k = 0.88$

Question 6, Page 491

Since cars are usually measured in metres, convert the millimetres to metres for each dimension provided.
The scale is 1:18

Height of 78.2 mm	Width of 93.5 mm	Length of 206.3 mm
$\frac{x \text{ m}}{78.2 \text{ mm}} = \frac{1 \text{ m}}{1000 \text{ mm}}$ $x \text{ m} = \frac{78.2 \text{ mm} \cdot 1 \text{ m}}{1000 \text{ mm}}$ $x \text{ m} = 0.0782 \text{ m}$ $h = \text{height of car}$ $k = \frac{\text{model}}{\text{actual}}$ $\frac{1}{18} = \frac{0.0782 \text{ m}}{h}$ $h = 18 \cdot 0.0782 \text{ m}$ $h = 1.4076 \text{ m}$	93.5 mm = 0.0935 m $w = \text{width of car}$ $k = \frac{\text{model}}{\text{actual}}$ $\frac{1}{18} = \frac{0.0935 \text{ m}}{w}$ $w = 18 \cdot 0.0935 \text{ m}$ $w = 1.683 \text{ m}$	206.3 mm = 0.2063 m $l = \text{length of car}$ $k = \frac{\text{model}}{\text{actual}}$ $\frac{1}{18} = \frac{0.2063 \text{ m}}{l}$ $l = 18 \cdot 0.2063 \text{ m}$ $l = 3.7134 \text{ m}$

Question 8, Page 491

a. Convert feet to inches.

$$\frac{x}{6.5 \text{ feet}} = \frac{12 \text{ in}}{1 \text{ ft}}$$

$$x = \frac{6.5 \text{ ft} \cdot 12 \text{ in}}{1 \text{ ft}}$$

$$x = 78 \text{ in}$$

$$k = \frac{\text{replica}}{\text{actual}}$$

$$k = \frac{26 \text{ in}}{78 \text{ in}}$$

$$k = \frac{1}{3}$$

The scale factor is $\frac{1}{3}$.

b.
$$\begin{aligned} \frac{x}{2.5\text{ ft}} &= \frac{12\text{ in}}{1\text{ ft}} \\ x &= \frac{2.5\text{ ft} \cdot 12\text{ in}}{1\text{ ft}} \\ x &= 30\text{ in} \end{aligned}$$

$$\begin{aligned} k &= \frac{\text{replica}}{\text{actual}} \\ \frac{1}{3} &= \frac{\text{replica}}{30\text{ in}} \\ \frac{1 \cdot 30\text{ in}}{3} &= \text{replica} \\ 10\text{ in} &= \text{replica} \end{aligned}$$

The replica models will be 10 inches wide.

Question 9, Page 491

Length	Width
$\frac{x}{32\text{ ft}} = \frac{12\text{ in}}{1\text{ ft}}$ $x = \frac{32\text{ ft} \cdot 12\text{ in}}{1\text{ ft}}$ $x = 384\text{ in}$ $k = \frac{\text{model}}{\text{actual}}$ $\frac{1}{24} = \frac{l}{384\text{ in}}$ $\frac{384\text{ in}}{24} = l$ $16\text{ inches} = l$	$k = \frac{\text{model}}{\text{actual}}$ $\frac{1}{24} = \frac{w}{48\text{ in}}$ $\frac{48\text{ in}}{24} = w$ $2\text{ inches} = w$

The scale model dimensions are 16 inches by 2 inches.

Question 11, Page 492

16.8 cm = 0.168 m $l = ?$ $k = \frac{\text{model}}{\text{actual}}$ $\frac{1}{200} = \frac{0.168\text{ m}}{l}$ $l = 200 \bullet 0.168\text{ m}$ $l = 33.6\text{ m}$	17.2 cm = 0.172 m wing span = ? $k = \frac{\text{model}}{\text{actual}}$ $\frac{1}{200} = \frac{0.172\text{ m}}{w}$ $w = 200 \bullet 0.172\text{ m}$ $w = 34.4\text{ m}$

Two passenger jets can fit in the 71.9 m wide by 46.6 m long Hangar 77. Two wing spans ($34.4 \times 2 = 68.8$ m) can fit the width of the building with 3.1 m to spare. The 46.6 m length of the hangar allows for only one row of jets, with 13 metres to spare at the end of each of the two jets parked side by side.

Question 13, Page 492

Height 45 cm h = height of table $k = \frac{\text{model}}{\text{actual}}$ $\frac{1}{10} = \frac{h}{45\text{ cm}}$ $\frac{45\text{ cm}}{10} = h$ $4.5\text{ cm} = h$	Width 50 cm w = width of table $k = \frac{\text{model}}{\text{actual}}$ $\frac{1}{10} = \frac{w}{50\text{ cm}}$ $\frac{50\text{ cm}}{10} = w$ $5.0\text{ cm} = w$	Length 100 cm l = length of table $k = \frac{\text{model}}{\text{actual}}$ $\frac{1}{10} = \frac{l}{100\text{ cm}}$ $\frac{100\text{ cm}}{10} = l$ $10.0\text{ cm} = l$

1 square = 1 cm