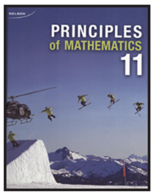
Lesson 5.3 Scale Factors, Areas, Surface Areas, and Volume



Refer to *Principles of Mathematics 11* pages 476-478 and 496-500 for more examples.

- Page 479, #1, 3, 5, 6, 10a, 11, 13
- Page 500, #1a, 1d, 2, 5a, 6, 9, 11, and 13

Question 1, Page 479

a. $k = \dfrac{\text{enlargement}}{\text{original}}$

$k = \dfrac{8\text{ cm}}{2\text{ cm}}$

$k = 4$

b. $\text{Area}_{\text{Rec }A} = 6 \times 2 = 12\text{ cm}^2$

$\text{Area}_{\text{Rec }B} = \text{Area}_{\text{Rec }A} \bullet k^2$

$\text{Area}_{\text{Rec }B} = 12\text{ cm}^2 \bullet (4)^2$

$\text{Area}_{\text{Rec }B} = 192\text{ cm}^2$

c. Let x = number of rectangle A that can fit in rectangle B

$x = \text{Area}_{\text{Rec }B} \div \text{Area}_{\text{Rec }A}$

$x = 192\text{ cm}^2 \div 12\text{ cm}^2$

$x = 16$

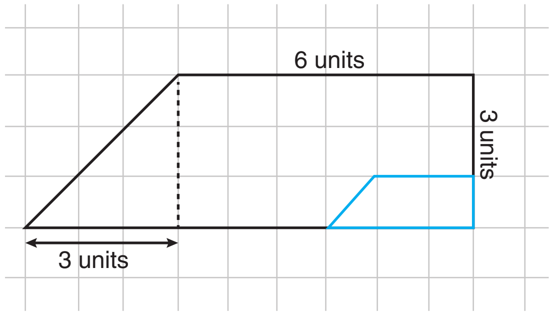
16 rectangle As can fit in one rectangle B.

Question 3, Page 479

$$\text{Area}_{\text{enlargement}} = \text{Area}_{\text{original}} \bullet k^2$$
$$\text{Area}_{\text{enlargement}} = 42\text{ cm}^2 \bullet (5)^2$$
$$\text{Area}_{\text{Rec }B} = 1050\text{ cm}^2$$

Question 5, Page 479

a.



$$A_{rec} = 6 \times 3 = 18 \text{ units}^2$$

$$A_{tri} = \frac{3 \times 3}{2} = 4.5 \text{ units}^2$$

$$\text{Total area} = 22.5 \text{ units}^2$$

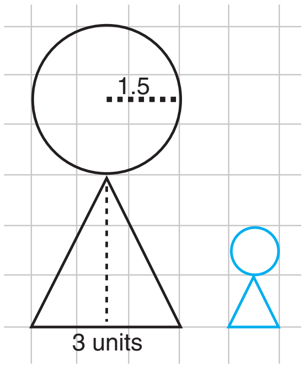
$$k = \frac{1}{3}$$

$$\text{Area}_{reduction} = \text{Area}_{original} \cdot k^2$$

$$\text{Area}_{reduction} = 22.5 \cdot \left(\frac{1}{3}\right)^2$$

$$\text{Area}_{reduction} = 2.5 \text{ units}^2$$

b.



$$\text{A}_{cir} = \pi(1.5)^2 \doteq 7.07 \text{ units}^2$$

$$\text{A}_{tri} = \frac{3 \times 3}{2} = 4.5 \text{ units}^2$$

$$\text{Total area} \doteq 11.57 \text{ units}^2$$

$$k = \frac{1}{3}$$

$$\text{Area}_{reduction} = \text{Area}_{original} \cdot k^2$$

$$\text{Area}_{reduction} = 11.57 \cdot \left(\frac{1}{3}\right)^2$$

$$\text{Area}_{reduction} \doteq 1.29 \text{ units}^2$$

Question 6, Page 480

a.

$k = \frac{\text{enlargment}}{\text{actual}}$	$k_{width} = \frac{\text{enlargment}}{\text{actual}}$	$k_{length} = \frac{\text{enlargment}}{\text{actual}}$
$k = \frac{150}{100}$	$1.5 = \frac{w}{4 \text{ in}}$	$1.5 = \frac{l}{6 \text{ in}}$
$k = 1.5$	$1.5 \cdot 4 \text{ in} = w$	$1.5 \cdot 6 \text{ in} = l$
	$6 \text{ in} = w$	$9 \text{ in} = l$

The inside dimensions of the frame are 6 inches by 9 inches.

b. When a shape is enlarged by a scale factor of k, the area of that shape is enlarged by a factor of $k^2 = (1.5)^2$.

$$k^2 = 2.25$$

$$2.25 \times 100\% = 225\%$$

The area of the photograph was increased by 225%.

c. Strategy #1: Input the dimensions of the enlarged photo (determined in part a) into the formula for the area of a rectangle
Strategy #2: Determine the area of the original digital photo and then multiply that area by the scale factor squared.

Question 10a, Page 480

$k = \frac{1}{120}$	$k = \frac{\text{display length}}{\text{actual length}}$ $\frac{1}{120} = \frac{3\text{ m}}{l}$ $l = 120 \bullet 3\text{ m}$ $l = 360\text{ m}$	$k = \frac{\text{display width}}{\text{actual width}}$ $\frac{1}{120} = \frac{2\text{ m}}{w}$ $w = 120 \bullet 2\text{ m}$ $w = 240\text{ m}$

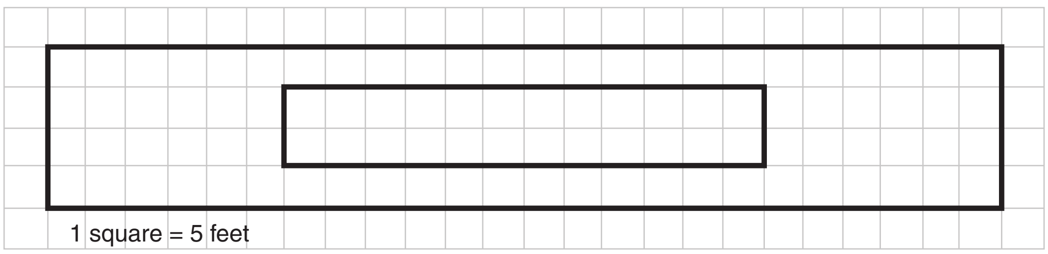
$$A_{\text{People's Park}} = 360 \times 240$$

$$A_{\text{People's Park}} = 86\,400\ m^2$$

Cost to maintain park $= 86400\text{ m}^2 \times \$0.75/\text{m}^2 = \$64\,800.00$

Question 11, Page 481

$k^2 = \frac{1}{4}$ $\sqrt{k^2} = \sqrt{\frac{1}{4}}$ $k = \frac{1}{2}$	$k = \frac{\text{mural length}}{\text{actual length}}$ $\frac{1}{2} = \frac{l}{120\text{ ft}}$ $\frac{1 \bullet 120\text{ ft}}{2} = l$ $60\text{ ft} = l$	$k = \frac{\text{mural height}}{\text{actual height}}$ $\frac{1}{2} = \frac{h}{20\text{ ft}}$ $\frac{1 \bullet 20\text{ ft}}{2} = h$ $10\text{ ft} = h$



Question 13

a. Area of enlargement = area of original • k^2

$$\frac{\text{Area of enlargement}}{\text{area of original}} = k^2$$

$$\frac{11.25 \text{ cm}^2}{5.00 \text{ cm}^2} = k^2$$

$$2.25 = k^2$$

$$\sqrt{2.25} = \sqrt{k^2}$$

$$1.5 = k$$

b. Area of reduction = area of original • k^2

$$\frac{\text{Area of reduction}}{\text{area of original}} = k^2$$

$$\frac{3.00 \text{ cm}^2}{12.00 \text{ cm}^2} = k^2$$

$$0.25 = k^2$$

$$\sqrt{0.25} = \sqrt{k^2}$$

$$0.5 = k$$

Question 1a, Page 500

$k = \frac{\text{enlargement}}{\text{actual}}$ $k = \frac{3 \text{ cm}}{1.5 \text{ cm}}$ $k = 2$	i. scale factor for area = k^2 $k^2 = 2^2 = 4$	ii. scale factor for volume = k^3 $k^3 = 2^3 = 8$

Question 1d, Page 500

$k = \frac{\text{enlargement}}{\text{actual}}$ $k = \frac{5 \text{ m}}{3 \text{ m}}$ $k = \frac{5}{3}$	i. scale factor for area = k^2 $k^2 = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$	ii. scale factor for volume = k^3 $k^3 = \left(\frac{5}{3}\right)^3 = \frac{125}{27}$

Question 2, Page 500

a. $k = \frac{\text{enlargment}}{\text{actual}}$

$k = \frac{600 \text{ mm}}{12 \text{ mm}}$

$k = 50$

b. scale factor for area $= k^2$

$k^2 = (50)^2 = 2500$

c. scale factor for volume $= k^3$

$k^3 = (50)^3 = 125000$

Question 5a, Page 501

$k = 3$

$$k^2 = \frac{\text{area of enlargement}}{\text{area of original}}$$

$$(3)^2 = \frac{\text{area of enlargement}}{500 \text{ cm}^2}$$

$$9 \cdot 500 = \text{area of enlargement}$$

$$4500 \text{ cm}^2 = \text{area of enlargement}$$

The area of each page in the large-print book is 4500 cm^2.

Question 6, Page 501

$$k = \frac{3}{4}$$

$$k^3 = \frac{\text{volume of reduction}}{\text{volume of original}}$$

$$\left(\frac{3}{4}\right)^3 = \frac{\text{volume of reduction}}{9420 \text{ cm}^3}$$

$$\left(\frac{3}{4}\right)^3 \cdot 9420 \text{ cm}^3 = \text{volume of reduction}$$

$$\left(\frac{27}{64}\right) \cdot 9420 \text{ cm}^3 = \text{volume of reduction}$$

$$3974.06 \text{ cm}^3 \doteq \text{volume of reduction}$$

The volume of the smaller vase is 3974.06 cm^3.

Question 9, Page 501

$k_{\text{doll}} = \frac{\text{enlargment}}{\text{actual}}$ $k = \frac{3.5\text{ cm}}{2\text{ cm}}$ $k = 1.75$	Determine the volume of the second smallest doll. $k = 1.75$ $k^3 = \frac{\text{volume of enlargment}}{\text{volume of original}}$ $(1.75)^3 = \frac{\text{volume of enlargment}}{8\text{ cm}^3}$ $(1.75)^3 \cdot 8\text{ cm}^3 = \text{volume of enlargment}$ $42.875\text{ cm}^3 = \text{volume of enlargment}$

1st doll: $V = 8\text{ cm}^3$
2nd doll: $V = 42.875\text{ cm}^3$
3rd doll: $V \doteq 42.875\text{ cm}^3 \times (1.75)^3 \doteq 229.8\text{ cm}^3$
4th doll: $V \doteq 229.8\text{ cm}^3 \times (1.75)^3 \doteq 1231.5\text{ cm}^3$
5th doll: $V \doteq 1231.5\text{ cm}^3 \times (1.75)^3 \doteq 6600\text{ cm}^3$

The volume of the largest doll can be estimated to be 6600 cm^3.

Question 11, Page 502

a.
$$k = \frac{\text{earth}}{\text{moon}}$$
$$k = \frac{6378.1\text{ km}}{1737.4\text{ km}}$$
$$k \doteq 3.67$$

$$k = \frac{\text{earth}}{\text{moon}}$$
$$3.67 = \frac{10.0\text{ cm}}{m}$$
$$3.67m = 10.0\text{ cm}$$
$$\frac{3.67}{3.67}m = \frac{10.0\text{ cm}}{3.67}$$
$$m \doteq 2.72\text{ cm}$$

The scale radius of the moon is 2.7 cm.

b. $3.67:1$

c. Use the square of the scale factor to compare surface areas.

$3.67^2:1^2 = 13.47:1$

d. Use the cube of the scale factor to compare the volumes.

$3.67^3:1^3 = 49.43:1$

Question 13, Page 502

$$k = \frac{3.8 \text{ in}}{2.9 \text{ in}}$$
$$k = 1.31$$

Use the square of the scale factor to compare surface areas.

$1.31^2 \doteq 1.72$

$1.72 = 172\%$

The softball requires 172% of the material of a baseball.
$172\% - 100\% = 72\%$ so the softball will need 72% more material than a baseball.