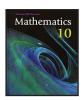


Enhance Your Understanding

Lesson 2.1: Surface Area of 3-D Objects



Refer to page 74 in Mathematics 10 for more practice.

• Page 74, #1, 3, 7, 11, and 12

Question 1, page 74

a.
$$SA_{prism} = 2lw + 2lh + 2wh$$

 $SA_{prism} = (2 \cdot 2.1 \text{ m} \cdot 1.2 \text{ m}) + (2 \cdot 2.1 \text{ m} \cdot 1.3 \text{ m}) + (2 \cdot 1.2 \text{ m} \cdot 1.3 \text{ m})$
 $SA_{prism} = 13.62 \text{ m}^2$

The surface area rounded to the nearest tenth is 13.6 m².

b. Radius is equal to half of the diameter.

$$r = \frac{d}{2} = \frac{38 \text{ in}}{2} = 19 \text{ in}$$

 $SA_{\text{cylinder}} = 2\pi r^2 + 2\pi rh$
 $SA_{\text{cylinder}} = 2\pi (19 \text{ in})^2 + (2\pi \cdot 19 \text{ in} \cdot 21 \text{ in})$
 $SA_{\text{cylinder}} = 4.775.220...\text{in}^2$

The surface area rounded to the nearest tenth is 4 775.2 in².

c.
$$SA_{cone} = \pi r^2 + \pi rs$$

 $SA_{cone} = \pi (4.5 \text{ cm})^2 + (2\pi \cdot 4.5 \text{ cm} \cdot 15 \text{ cm})$
 $SA_{cone} = 275.674... \text{ cm}^2$

The surface area rounded to the nearest tenth is 275.7 cm².

d.
$$SA_{\text{pyramid}} = lw + 2\left(\frac{1}{2}ls_1\right) + \frac{1}{2}ws_2$$

 $SA_{\text{pyramid}} = (10 \text{ in} \cdot 8 \text{ in}) + 2\left(\frac{1}{2}(10 \text{ in})(8.5 \text{ in})\right) + \left(\frac{1}{2} \cdot 8 \text{ in} \cdot 9 \text{ in}\right)$
 $SA_{\text{pyramid}} = 237 \text{ in}^2$

The surface area is 237 in².

e.
$$SA_{\text{sphere}} = 4\pi r^2$$

 $SA_{\text{sphere}} = 4\pi (3.6 \text{ cm})^2$
 $SA_{\text{sphere}} = 162.860... \text{ cm}^2$

The surface area rounded to the nearest tenth is 162.9 cm².

Question 3, page 74

a. First, substitute the given surface area and dimensions into the formula below. Then, solve for the missing dimension.

$$SA_{\text{prism}} = 2lw + 2lh + 2wh$$

$$6020 \text{ cm}^2 = (2 \cdot 34 \text{ cm} \cdot w) + (2 \cdot 34 \text{ cm} \cdot 20 \text{ cm}) + (2 \cdot w \cdot 20 \text{ cm})$$

$$6020 \text{ cm}^2 = 68 \text{ cm} \cdot w + 1360 \text{ cm}^2 + 40 \text{ cm} \cdot w$$

$$6020 \text{ cm}^2 - 1360 \text{ cm}^2 = 108 \text{ cm} \cdot w + 1360 \text{ cm}^2 - 1360 \text{ cm}^2$$

$$\frac{4460 \text{ cm}^2}{108 \text{ cm}} = \frac{108 \text{ cm}}{108 \text{ cm}} \cdot w$$

$$43.148...\text{cm} = w$$

The missing width rounded to the nearest tenth is 43.1 cm.

b. First, substitute the given surface area and dimensions into the formula below. Then, solve for the missing dimension.

$$SA_{\text{cylinder}} = 2\pi r^2 + 2\pi rh$$

$$47.4 \text{ m}^2 = 2\pi (0.65 \text{ m})^2 + (2\pi \cdot 0.65 \text{ m} \cdot h)$$

$$47.4 \text{ m}^2 - 2\pi (0.65 \text{ m})^2 = 2\pi (0.65 \text{ m})^2 - 2\pi (0.65 \text{ m})^2 + (2\pi \cdot 0.65 \text{ m} \cdot h)$$

$$\frac{47.4 \text{ m}^2 - 2\pi (0.65 \text{ m})^2}{2\pi \cdot 0.65 \text{ m}} = \frac{2\pi \cdot 0.65 \text{ m}}{2\pi \cdot 0.65 \text{ m}} \cdot h$$

$$10.956... \text{ m} = h$$

The missing height rounded to the nearest tenth is 11.0 m.

c. First, substitute the given surface area and dimensions into the formula below. Then, solve for the missing dimension.

$$SA_{\text{sphere}} = 4\pi r^2$$
 $91.4 \text{ m}^2 = 4\pi r^2$
 $\frac{91.4 \text{ m}^2}{4\pi} = \frac{4\pi}{4\pi} r^2$
 $\sqrt{\frac{91.4 \text{ m}^2}{4\pi}} = \sqrt{r^2}$
 $r = 2.696... \text{ m}$

The missing radius rounded to the nearest tenth is 2.7 m.

Question 7, page 75

A tipi with a fabric floor is shaped like a cone so use the formula below.

$$SA_{\text{tipi}} = \pi r^2 + \pi rs$$

 $SA_{\text{tipi}} = \pi (4.8 \text{ m})^2 + (\pi \cdot 4.8 \text{ m} \cdot 7.3 \text{ m})$
 $SA_{\text{tipi}} = 182.463... \text{ m}^2$

The minimum amount of canvas needed for the sides of the tipi is 182.46 m², rounded to the nearest hundredth.

Question 11, page 75

To calculate surface area, you need the radius.

diameter,
$$d = 2r$$

$$14\frac{7}{8} \text{ in } = 2r$$

$$\frac{14.875 \text{ in}}{2} = \frac{2}{2}r$$

$$7.4375 \text{ in } = r$$

$$SA_{\text{Drum}} = \text{Top} + \text{Lateral Surfaces}$$

 $SA = \pi r^2 + 2\pi rh$
 $SA = \pi (7.4375 \text{ in})^2 + (2\pi \cdot 7.4375 \text{ in} \cdot 3 \text{ in})$
 $SA = 313.975... \text{ in}^2$

The minimum amount of hide needed to make the drum is 314 in², rounded to the nearest square inch.

Question 12, page 77

To determine how much glass is needed for each of the large greenhouse, you need to find the area of the 4 sides (triangles).

$$SA = 4 \cdot \left(\frac{1}{2}bh\right)$$

$$SA = 4 \cdot \left(\frac{1}{2} \cdot 26 \text{ m} \cdot 35.4 \text{ m}\right)$$

$$SA \doteq 1840.8 \text{ m}^2$$

Each large greenhouse needs 1841 m² of glass, rounded to the nearest square metre.

Lesson 2.2: Volume of 3-D Objects



Refer to page 86 in Mathematics 10 for more practice.

• Page 86, #1, 2, and 5

Question 1, page 86

a. First, identify the length, width, and height: l = 85 cm, w = 68 cm, h = 1 m. Since height is in a different unit, convert it from metres to centimeters using 1 m = 100 cm. Then, find the volume using the formula:

$$V_{\text{pyramid}} = \frac{1}{3} lwh$$

$$V_{\text{pyramid}} = \frac{1}{3} \cdot 85 \text{ cm} \cdot 68 \text{ cm} \cdot 100 \text{ cm}$$

$$V_{\text{pyramid}} = 192 666.666...\text{cm}^3$$

The volume of the pyramid is 192 666.7 cm³ when rounded to the nearest tenth.

b. First, identify the length, width, and height: l = 85 cm, w = 68 cm, h = 1 m. Since height is in a different unit, convert it from metres to centimeters using 1 m = 100 cm. Then, find the volume using the formula:

$$V_{
m rectangular \, prism} = lwh$$

 $V_{
m rectangular \, prism} = 85 \, {\rm cm \cdot 68 \, cm \cdot 100 \, cm}$
 $V_{
m rectangular \, prism} = 578 \, 000 \, {\rm cm}^3$

The volume of the rectangular prism is 578 000 cm³.

c. First, identify the radius and height: r = 4 ft, h = 20 ft. Then, find the volume using the formula:

$$V_{\text{cylinder}} = \pi r^2 h$$

$$V_{\text{cylinder}} = \pi (4 \text{ ft})^2 \cdot 20 \text{ ft}$$

$$V_{\text{cylinder}} = 1005.309... \text{ ft}^3$$

The volume of the cylinder is 1005.3 ft³ when rounded to the nearest tenth.

d. First, identify the radius and height: r = 4 ft, h = 20 ft. Then, find the volume using the formula:

$$V_{\rm cone} = \frac{1}{3}\pi r^2 h$$

$$V_{\text{cone}} = \frac{1}{3}\pi (4 \text{ ft})^2 \cdot 20 \text{ ft}$$

$$V_{\text{cone}} = 335.103... \text{ ft}^3$$

The volume of the cone is 355.1 ft³ when rounded to the nearest tenth.

e. First, identify the radius: r = 81 mm. Then, find the volume using the formula:

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3$$

$$V_{\rm sphere} = \frac{4}{3}\pi (81 \text{ mm})^3$$

$$V_{\text{sphere}} = 2\ 226\ 094.855...\ \text{mm}^3$$

The volume of the sphere is 2 226 094.9 mm³ when rounded to the nearest tenth.

Question 2, page 86

a. First, identify the length, width, and height: l = 6 in, w = 6 in, h = 4 in. Then, find the volume using the formula:

$$V_{\text{pyramid}} = \frac{1}{3}lwh$$

$$V_{\text{pyramid}} = \frac{1}{3} \cdot 6 \text{ in} \cdot 6 \text{ in} \cdot 4 \text{ in}$$

$$V_{\text{pyramid}} = 48 \text{ in}^3$$

The volume of the pyramid is 48 in³.

b. First, identify the radius and height: r = 12 cm, h = 5 cm. Then, find the volume using the formula:

$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h$$

 $V_{\text{cone}} = \frac{1}{3}\pi (12 \text{ cm})^2 \cdot 5 \text{ cm}$
 $V_{\text{cone}} = 753.982... \text{ cm}^3$

The volume of the cone is 754.0 cm³ when rounded to the nearest tenth.

Question 5, page 87

a. First, substitute the given volume and dimensions into the formula below. Then, solve for the missing dimension.

$$V_{\text{rectangular prism}} = lwh$$

$$161.6 \text{ cm}^3 = (5.2 \text{ cm} \cdot 3.7 \text{ cm})h$$

$$161.6 \text{ cm}^3 = 19.24 \text{ cm}^2 \cdot h$$

$$\frac{161.6 \text{ cm}^3}{19.24 \text{ cm}^2} = \frac{19.24 \text{ cm}^2}{19.24 \text{ cm}^2} \cdot h$$

$$8.399... \text{ cm} = h$$

The height of the rectangular prism is 8.4 cm when rounded to the nearest tenth.

b. First, substitute the given volume and dimensions into the formula below. Then, solve for the missing dimension.

$$V_{\text{pyramid}} = \frac{1}{3} lwh$$

$$196 \text{ in}^3 = \frac{1}{3} (7 \text{ in} \cdot 7 \text{ in}) \cdot h$$

$$196 \text{ in}^3 = \frac{1}{3} (49 \text{ in}^2) \cdot h$$

$$196 \text{ in}^3 \cdot 3 = \frac{1}{3} \cdot 3 (49 \text{ in}^2) \cdot h$$

$$\frac{588 \text{ in}^3}{49 \text{ in}^2} = \frac{49 \text{ in}^2}{49 \text{ in}^2} \cdot h$$

$$12 \text{ in} = h$$

The height of the pyramid is 12 inches.

c. First, substitute the given volume and dimensions into the formula below. Then, solve for the missing dimension.

$$V_{\text{cylinder}} = \pi r^{2} h$$

$$339.3 \text{ cm}^{3} = \pi \cdot r^{2} \cdot 3 \text{ cm}$$

$$\frac{339.3 \text{ cm}^{3}}{\pi \cdot 3 \text{ cm}} = \frac{\pi \cdot 3 \text{ cm}}{\pi \cdot 3 \text{ cm}} \cdot r^{2}$$

$$\frac{339.3 \text{ cm}^{3}}{\pi \cdot 3 \text{ cm}} = r^{2}$$

$$\sqrt{\frac{339.3 \text{ cm}^{3}}{\pi \cdot 3 \text{ cm}}} = \sqrt{r^{2}}$$

$$6.005... \text{ cm} = r$$

The radius of the cylinder is 6.0 cm when rounded to the nearest tenth.

d. First, substitute the given volume and dimensions into the formula below. Then, solve for the missing dimension.

$$V_{\text{cone}} = \frac{1}{3}\pi r^{2}h$$

$$8.1 \text{ yd}^{3} = \frac{1}{3}\pi r^{2} \cdot 2.4 \text{ yd}$$

$$8.1 \text{ yd}^{3} = \left(\frac{1}{3} \cdot 2.4 \text{ yd}\right)\pi \cdot r^{2}$$

$$8.1 \text{ yd}^{3} = (0.8 \text{ yd})\pi \cdot r^{2}$$

$$\frac{8.1 \text{ yd}^{3}}{(0.8 \text{ yd})\pi} = \frac{(0.8 \text{ yd})\pi}{(0.8 \text{ yd})\pi} r^{2}$$

$$\frac{8.1 \text{ yd}^{3}}{(0.8 \text{ yd})\pi} = r^{2}$$

$$\sqrt{\frac{8.1 \text{ yd}^{3}}{(0.8 \text{ yd})\pi}} = \sqrt{r^{2}}$$

$$1.795... \text{ yd} = r$$

The radius of the cone is 1.8 yd when rounded to the nearest tenth.

Lesson 2.3: Composite Objects Applications



Refer to pages 74 and 86 in *Mathematics 10* for more practice.

- Page 74, #5, 6, 14, 15, and 16 Page 86, #3, 4, 6, 7, 11, 12, 14

Question 5, page 75

The surface area of the composite object is equal to the surface areas of the pyramid and the sphere.

For the pyramid:

$$SA_{\text{pyramid}} = lw + 2\left(\frac{1}{2}ls_1\right) + \frac{1}{2}ws_2$$

 $SA_{\text{pyramid}} = (15 \text{ cm} \cdot 12 \text{ cm}) + 2\left(\frac{1}{2} \cdot 15 \text{ cm} \cdot 12.75 \text{ cm}\right) + \left(\frac{1}{2} \cdot 12 \text{ cm} \cdot 13.5 \text{ cm}\right)$
 $SA_{\text{pyramid}} = 533.25 \text{ cm}^2$

For the sphere:

$$SA_{\text{sphere}} = 4\pi r^2$$

 $SA_{\text{sphere}} = 4\pi (4 \text{ cm})^2$
 $SA_{\text{sphere}} = 201.061... \text{ cm}^2$
 $SA_{\text{Composed Object}} = SA_{\text{pyramid}} + SA_{\text{sphere}}$
 $= 533.25 \text{ cm}^2 + 201.061... \text{ cm}^2$
 $= 734.311... \text{ cm}^2$

The surface area of the composite object is approximately 734.3 cm².

Question 6, page 75

There are no errors in Austin's calculations. It is better to not round until the last step; in this case it only makes a tiny difference.

For four cylinders:

$$SA = 4(16.5\pi)$$

 $SA = 207.345...$ ft²

The surface area is 207.35 ft², to the nearest hundredth.

He does not need to paint the bottom ends of the pillars, as they will be on the ground. In this case,

$$SA = 4(\pi r^2 + \pi dh)$$

$$SA = 4(\pi (0.5 \text{ ft})^2 + (\pi \cdot 1 \text{ ft} \cdot 165 \text{ ft}))$$

$$SA = 204.203...\text{ft}^2$$

The surface area you would paint is approximately 204.20 ft².

Question 14, page 77

Surface area of the light tent is equal the area of the cylindrical wall and the conical roof.

For the cylindrical wall:

$$SA = 2\pi rh$$
 which can be rewritten as $SA = \pi dh$ since $2r = d$

$$SA = \pi (1 \text{ m})(1.35 \text{ m})$$

$$SA = 1.35\pi \text{ m}^2$$

For the conical roof:

To determine the slant height, s, of the roof, use the Pythagorean theorem.

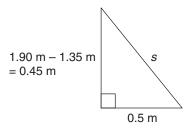
$$s^{2} = (0.5 \text{ m})^{2} + (0.45 \text{ m})^{2}$$

$$s^{2} = 0.25 \text{ m}^{2} + 0.2025 \text{ m}^{2}$$

$$s^{2} = 0.4525 \text{ m}^{2}$$

$$\sqrt{s^{2}} = \sqrt{0.4525 \text{ m}^{2}}$$

$$s = \sqrt{0.4525 \text{ m}^{2}}$$



$$SA = \pi rs$$

$$SA = \pi \cdot 0.5 \text{ m} \cdot \sqrt{0.4525 \text{ m}^2}$$

$$SA_{\text{light tent}} = \text{wall} + \text{roof}$$

= 1.35 π m² + ($\pi \cdot 0.5$ m · $\sqrt{0.4525}$ m²)
= 5.29... m²

The surface area of the light tent is 5 m² when rounded to the nearest square metre.

Question 15, page 78

a. With the minimum diameter of 39.5 mm:

$$r = \frac{d}{2}$$

$$r = \frac{39.5 \text{ mm}}{2}$$

$$r = 19.75 \text{ mm}$$

$$SA_{\min} = 4\pi r^2$$

$$SA_{\min} = 4\pi (19.75 \text{ mm})^2$$

$$SA_{min} = 4901.669... \text{ mm}^2$$

With the maximum diameter of 40.5 mm:

$$r = \frac{d}{2}$$

$$r = \frac{40.5 \text{ mm}}{2}$$

$$r = 20.25 \text{ mm}$$

$$SA_{min} = 4\pi r^2$$

 $SA_{min} = 4\pi (20.25 \text{ mm})^2$
 $SA_{min} = 5152.997... \text{ mm}^2$

For a regulation squash ball the minimum surface area is approximately 4092 mm² and the maximum is 5153 mm².

b. For the box, the minimum side length is 39.5 mm and the maximum is 40.5 mm.

Question 16, page 78

To determine the amount of paper in the funnel, you can subtract the surface area of the cut off part from the surface area of the whole paper cone.

For the whole paper cone:

$$SA = \pi rs$$

$$SA = \pi \cdot 4.75 \text{ cm} \cdot 13.2 \text{ cm}$$

$$SA = 62.7\pi \text{ cm}^2$$

For the cut off part:

$$SA = \pi rs$$

$$SA = \pi \cdot 0.75 \text{ cm} \cdot 2 \text{ cm}$$

$$SA = 1.5\pi \text{ cm}^2$$

Amount of paper in funnel =
$$62.7\pi$$
 cm² - 1.5π cm²
= 192.265 ... cm²

The amount of paper in the funnel is approximately 192.3 cm².

Question 3, page 86

To calculate the volume of the object in cubic metres, convert all given dimensions from centimetres to metres using 1 cm = 0.01 m.

To calculate the volume of a cylinder, you need the radius.

diameter,
$$d = 2r$$

 $0.064 = 2r$
 $0.032 = r$

Total Volume = Volume of Rectangular Prism Base + Volume of 3 cylinders
=
$$lwh + 3 \cdot \pi r^2 h$$

= $(0.75 \text{ m} \cdot 0.858 \text{ m} \cdot 0.137 \text{ m}) + 3(\pi (0.032 \text{ m})^2 \cdot 0.311 \text{ m})$
= $0.091 \ 116...\text{m}^3$

The volume of the composite object is 0.1 m³ when rounded to the nearest tenth.

Question 4, page 87

- a. Erin calculated the volume correctly. Janine made an error in the first step. The fraction should only multiply the first $\pi r^2 h$, to give the volume of the conical top part.
- b. Erin's method should be used.

Question 6, page 87

Since the question requires an answer expressed in cubic metres, you should convert all dimensions to metres using 1 km = 1000 m and 1 m = 1000 mm.

Length of the longest section of the pipeline = $1221.73 \text{ km} \cdot \frac{1000 \text{ m}}{1 \text{ km}} = 1221730 \text{ m}$

Diameter of the longest section of the pipeline = 914 mm $\cdot \frac{1 \text{ m}}{1000 \text{ mm}} = 0.914 \text{ m}$

$$V = \pi r^2 h$$

$$V = \pi (0.914 \text{ m})^2 \cdot 1221730 \text{ m}$$

$$V = 801599.635...\text{m}^3$$

This section of the pipeline can hold approximately 801 599.64 m³ of oil.

Question 7, page 88

First, determine the volume of the rectangular prism. Then substitute the volume into the formula for volume of a cube to find the dimensions.

$$V_{\text{Rectangular Prism}} = lwh$$

$$= 9 \text{ in } \cdot 4 \text{ in } \cdot 6 \text{ in}$$

$$= 216 \text{ in}^{3}$$

$$V_{\text{Cube}} = s^{3}$$

$$216 \text{ in}^{3} = s^{3}$$

$$\sqrt[3]{216 \text{ in}^{3}} = \sqrt[3]{s^{3}}$$

$$6 \text{ in } = s$$

The cube would have side length of 6 inches to have the same volume as the rectangular prism.

Question 11, page 86

Volume of Snow = Volume of Hemisphere – Inside Volume – Ventilation Hole
$$= \frac{1}{2} \left(\frac{4}{3} \pi r^3 \right) - \frac{1}{2} \left(\frac{4}{3} \pi r^3 \right) - 0.0068 \text{ m}^3$$
$$= \frac{1}{2} \left(\frac{4}{3} \pi (1.8 \text{ m})^3 \right) - \frac{1}{2} \left(\frac{4}{3} \pi (1.5 \text{ m})^3 \right) - 0.0068 \text{ m}^3$$
$$= 5.139...\text{m}^3$$

Approximately, 5.1 m³ was used to construct the igloo.

Question 12, page 89

First, substitute the given volume and dimensions into the formula below. Then, solve for the missing height, h.

$$V_{\text{pyramid}} = \frac{1}{3} lwh$$

$$4096 \text{ ft}^3 = \frac{1}{3} (32 \text{ ft} \cdot 32 \text{ ft}) \cdot h$$

$$4096 \text{ ft}^3 = \frac{1}{3} (1024 \text{ ft}^2) \cdot h$$

$$1024 \text{ ft}^3 \cdot 3 = \frac{1}{3} \cdot 3 (1024 \text{ ft}^2) \cdot h$$

$$4096 \text{ ft}^3 = 1024 \text{ ft}^2 \cdot h$$

$$\frac{4096 \text{ ft}^3}{1024 \text{ ft}^2} = \frac{1024 \text{ ft}^2}{1024 \text{ ft}^2} \cdot h$$

$$12 \text{ ft} = h$$

The minimum height for the apex of the roof is 12 ft.

Question 14, page 86

Based on the description and diagram of the souvenir piece, the total volume is equal to three quarters of the cube and the entire sphere.

First, determine the volume of the cube. Since one quarter of the cube is replaced by part of the sphere, three quarters of the cube remains.

Volume of Cube =
$$s^3$$

= $(4 \text{ cm})^3$
= 64 cm^3

Volume of Cube that remains =
$$\frac{3}{4}$$
(64 cm³)
= 48 cm³

Volume of Sphere =
$$\frac{4}{3}\pi r^3$$

= $\frac{4}{3}\pi (2 \text{ cm})^3$
= 33.510...cm³

Total Volume =
$$48 \text{ cm}^3 + 33.510...\text{cm}^3$$

= $81.510...\text{cm}^3$

The volume of the souvenir is approximately 81.5 cm³.