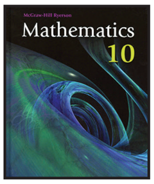
Enhance Your Understanding

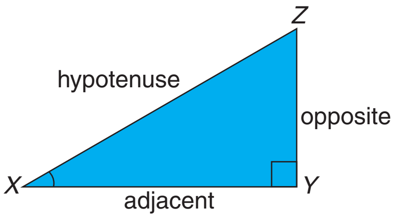
Lesson 3.1: The Tangent Ratio



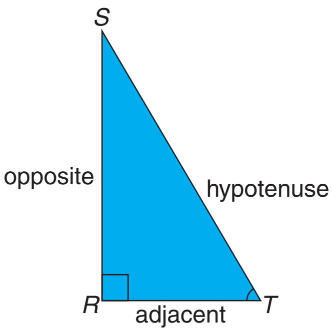
Refer to page 107 in *Mathematics 10* for more practice.
- Page 107, #1, 3a, 3b, 4a, 4b, 5, 6, 8, 9, 13

Question 1, page 107

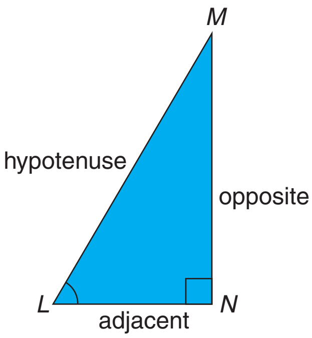
a.



b.



c.



Question 3, page 108

a. $\tan 74^\circ = 3.487\ 414...$

$\tan 74^\circ \doteq 3.4874$, rounded to 4 decimal places

b. $\tan 45^\circ = 1$

Question 4, page 108

a. $\tan A = 0.7$

$\angle A = \tan^{-1}(0.7)$

$\angle A = 34.992...°$

$\angle A \doteq 35°$

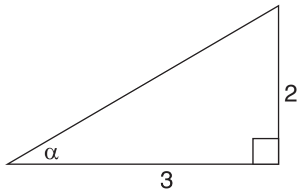
b. $\tan \theta = 1.75$

$\theta = \tan^{-1}(1.75)$

$\theta = 60.255...°$

$\theta \doteq 60°$

Question 5, page 108

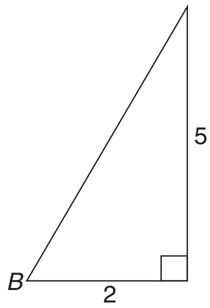


a. Since $\tan \alpha = \frac{2}{3}$, the side opposite the angle α is labelled 2 and the side adjacent to the angle α is labelled 3. To calculate the angle α, apply the inverse tangent.

$\alpha = \tan^{-1}\left(\frac{2}{3}\right)$

$\alpha = 33.690...°$

$\alpha \doteq 34°$



b. Since $\tan B = \frac{5}{2}$, the side opposite the angle B is labelled 5 and the side adjacent to the angle B is labelled 2. To calculate the angle B, apply the inverse tangent.

$\angle B = \tan^{-1}\left(\frac{5}{2}\right)$

$\angle B = 68.198...°$

$\angle B \doteq 68°$

Question 6, page 108

a.
$$\tan 33° = \frac{\text{length opposite } 33°}{\text{length adjacent to } 33°}$$

$$\tan 33° = \frac{x}{30.5 \text{ m}}$$

$$30.5 \text{ m} \cdot \tan 33° = \frac{x}{\cancel{30.5 \text{ m}}} \cdot \cancel{30.5 \text{ m}}$$

$$19.806... \text{m} = x$$

$$19.8 \text{ m} \doteq x$$

The height of the tree is approximately 19.8 m.

b. $\tan\theta = \dfrac{\text{length opposite } \theta}{\text{length adjacent to } \theta}$

$\tan\theta = \dfrac{1.25}{20}$

$\theta = \tan^{-1}\left(\dfrac{1.25}{20}\right)$

$\theta = 3.576...^\circ$

$\theta \doteq 3.6^\circ$

The measure of angle θ is approximately 3.6˚.

Question 8, page 108

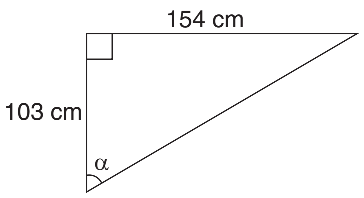
Assume that the flag is rectangular. As such, the triangle contains one right angle. Let α represent the larger acute angle in the triangle containing the fleur-de-lys.

$\tan\alpha = \dfrac{\text{length opposite } \alpha}{\text{length adjacent to } \alpha}$

$\tan\alpha = \dfrac{154}{103}$

$\alpha = \tan^{-1}\left(\dfrac{154}{103}\right)$

$\alpha = 56.224...^\circ$



The larger acute angle is approximately 56˚. Since the interior angles in any triangle must add up to 180˚, we can subtract 90˚ and 56˚ from 180˚ to determine the measure of the other acute angle.

$180^\circ - 90^\circ - 56^\circ = 34^\circ$

The angles of the triangle containing the fleur-de-lys are 56˚, 90˚, and 34˚.

Question 9, page 109

a. $\tan 6^\circ = \dfrac{\text{length opposite } 6^\circ}{\text{length adjacent to } 6^\circ}$

$\tan 6^\circ = \dfrac{3}{x}$

$x \bullet \tan 6^\circ = \dfrac{3}{\cancel{x}} \bullet \cancel{x}$

$\dfrac{x \bullet \cancel{\tan 6^\circ}}{\cancel{\tan 6^\circ}} = \dfrac{3}{\tan 6^\circ}$

$x = 28.543...$

The horizontal length of the ramp is approximately 29 ft.

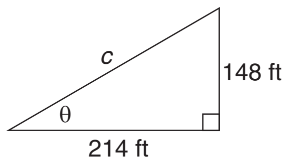
b. Use a proportion to determine the minimum acceptable horizontal length of a ramp with a rise of 3 feet.

$$\frac{1}{12} = \frac{3}{x}$$
$$x = 3 \cdot 12$$
$$x = 36$$

A horizontal length of 36 ft is needed for a ramp with a rise of 3 feet. Since 29 feet is less than 36 feet, the ramp shown would not be considered safe.

Question 13, page 110

a. Let θ represent the angle of steepness of the hill



$$\tan\theta = \frac{\text{length opposite } \theta}{\text{length adjacent to } \theta}$$
$$\tan\theta = \frac{148}{214}$$
$$\theta = \tan^{-1}\left(\frac{148}{214}\right)$$
$$\theta = 34.667...^\circ$$

The angle of elevation of the hill is approximately 35˚.

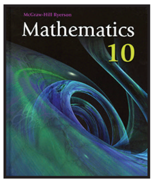
b. Let *c* represent the hypotenuse, the distance up the hill.

Use the Pythagorean theorem.

$$c^2 = a^2 + b^2$$
$$c^2 = 214^2 + 148^2$$
$$c^2 = 67700$$
$$c = \sqrt{67700}$$
$$c = 260.192...$$

The distance you would have to climb to the top of the hill is approximately 260 ft.

Lesson 3.2 The Sine and Cosine Ratios



Refer to page 120 in *Mathematics 10* for more practice.
- Page 120, #1a, 1c, 2a, 2c, 3a, 3c, 4, 5, 6, 9, 11, 12

Question 1, page 120

a. $\cos 34° = 0.829\ 037...$

$\cos 34° \doteq 0.8290$, rounded to 4 decimal places

c. $\sin 62.9° = 0.890\ 212...$

$\sin 62.9° \doteq 0.8902$, rounded to 4 decimal places

Question 2, page 120

a. $\sin A = \dfrac{\text{length opposite } \angle A}{\text{hypotenuse}}$

$\sin A = \dfrac{24}{26}$

$\sin A = \dfrac{12}{13}$

c. $\cos C = \dfrac{\text{length adjacent to } \angle C}{\text{hypotenuse}}$

$\cos C = \dfrac{24}{26}$

$\cos C = \dfrac{12}{13}$

Question 3, page 120

a. $\cos A = 0.4621$

$\angle A = \cos^{-1}(0.4621)$

$\angle A \doteq 62°$

c. $\sin \beta = 0.5543$

$\beta = \sin^{-1}(0.5543)$

$\beta \doteq 34°$

Question 4, page 120

a. $\cos 40° = \dfrac{\text{length adjacent to } 40°}{\text{hypotenuse}}$

$\cos 40° = \dfrac{x}{20}$

$20 \bullet \cos 40° = \dfrac{x}{\cancel{20}} \bullet \cancel{20}$

$15.3 \doteq x$

b. $\sin 18^\circ = \dfrac{\text{length opposite } 18^\circ}{\text{hypotenuse}}$

$x \cdot \sin 18^\circ = \dfrac{7}{\cancel{x}} \cdot \cancel{x}$

$\dfrac{x \cdot \cancel{\sin 18^\circ}}{\cancel{\sin 18^\circ}} = \dfrac{7}{\sin 18^\circ}$

$x \doteq 22.7$

Question 5, page 120

a. $\sin\theta = \dfrac{\text{length opposite } \theta}{\text{hypotenuse}}$

$\sin\theta = \dfrac{7}{12.8}$

$\theta = \sin^{-1}\left(\dfrac{7}{12.8}\right)$

$\theta \doteq 33.2^\circ$

b. $\cos\theta = \dfrac{\text{length adjacent to } \theta}{\text{hypotenuse}}$

$\cos\theta = \dfrac{16}{20}$

$\theta = \cos^{-1}\left(\dfrac{16}{20}\right)$

$\theta \doteq 36.9^\circ$

Question 6, page 121

a. $\sin\theta = \dfrac{\text{length opposite } \theta}{\text{hypotenuse}}$

$\sin\theta = \dfrac{7}{10}$

$\theta = \sin^{-1}\left(\dfrac{7}{10}\right)$

$\theta \doteq 44.4^\circ$

b. $\cos 65^\circ = \dfrac{\text{length adjacent to } 65^\circ}{\text{hypotenuse}}$

$\cos 65^\circ = \dfrac{6}{x}$

$x \cdot \cos 65^\circ = \dfrac{6}{\cancel{x}} \cdot \cancel{x}$

$\dfrac{x \cdot \cancel{\cos 65^\circ}}{\cancel{\cos 65^\circ}} = \dfrac{6}{\cos 65^\circ}$

$x \doteq 14.2 \text{ ft}$

c. $\cos\theta = \dfrac{\text{length adjacent to } \theta}{\text{hypotenuse}}$

$\cos\theta = \dfrac{100}{132}$

$\theta = \cos^{-1}\left(\dfrac{100}{132}\right)$

$\theta \doteq 40.7^\circ$

d. $\sin 18^\circ = \dfrac{\text{length opposite } 18^\circ}{\text{hypotenuse}}$

$\sin 18^\circ = \dfrac{70}{x}$

$x \cdot \sin 18^\circ = \dfrac{70}{\cancel{x}} \cdot \cancel{x}$

$\dfrac{x \cdot \cancel{\sin 18^\circ}}{\cancel{\sin 18^\circ}} = \dfrac{70}{\sin 18^\circ}$

$x \doteq 226.5 \text{ m}$

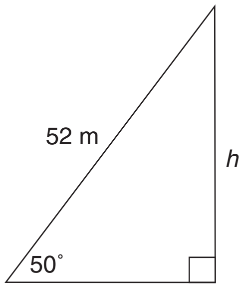
Question 9, page 122
Let *h* represents the height of the oil rig.

$\sin 50^\circ = \dfrac{\text{length opposite } 50^\circ}{\text{hypotenuse}}$

$\sin 50^\circ = \dfrac{h}{52 \text{ m}}$

$52 \cdot \sin 50^\circ = \dfrac{h}{\cancel{52 \text{ m}}} \cdot \cancel{52 \text{ m}}$

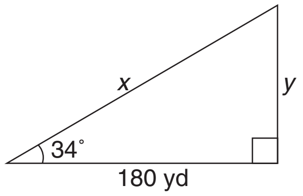
$39.834... \text{ m} = h$



The height of the oil rig is approximately 39.8 m.

Question 11, page 122

a. Let x represent the direct distance from the teeing area to the flag and let y represent the third side of the right triangle as shown.



b.
$$\cos 34^\circ = \frac{\text{length adjacent to } 34^\circ}{\text{hypotenuse}}$$
$$\cos 34^\circ = \frac{180 \text{ yd}}{x}$$
$$x \bullet \cos 34^\circ = \frac{180 \text{ yd}}{\cancel{x}} \bullet \cancel{x}$$
$$\frac{x \bullet \cancel{\cos 34^\circ}}{\cancel{\cos 34^\circ}} = \frac{180 \text{ yd}}{\cos 34^\circ}$$
$$x \doteq 217 \text{ yd}$$

The direct distance from the teeing area to the flag is approximately 217 yd.

c.
$$\tan 34^\circ = \frac{\text{length opposite } 34^\circ}{\text{length adjacent to } 34^\circ}$$
$$\tan 34^\circ = \frac{y}{180 \text{ yd}}$$
$$180 \text{ yd} \bullet \tan 34^\circ = \frac{y}{\cancel{180 \text{ yd}}} \bullet \cancel{180 \text{ yd}}$$
$$121 \text{ yd} \doteq y$$

The other leg of the track measures approximately 121 yd. So, the distance along the track is approximately $180 + 121 = 301$ yards.

$$\begin{aligned}\text{track distance} - \text{direct distance} &= 301 \text{ yd} - 217 \text{ yd}\\ &= 84 \text{ yd}\end{aligned}$$

The direct distance is approximately 84 yards shorter than following the track.

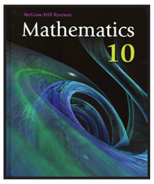
Question 12, page 122

Let θ represent the angle between the cable and the horizontal.

$$\cos\theta = \frac{\text{length adjacent to } \theta}{\text{hypotenuse}}$$
$$\cos\theta = \frac{2200 \text{ m}}{2300 \text{ m}}$$
$$\theta = \cos^{-1}\left(\frac{2200 \text{ m}}{2300 \text{ m}}\right)$$
$$\theta \doteq 17^\circ$$

The angle that the line of sag makes with the horizontal is approximately 17˚.

Lesson 3.3 Solving Problems with Triangles



Refer to page 131 in *Mathematics 10* for more practice.

- Page 131, #1a, 1c, 2, 3, 4, 7, 11, 12

Question 1, page 131

a.

$$\cos 30^\circ = \frac{\text{length adjacent to } 30^\circ}{\text{hypotenuse}}$$
$$\cos 30^\circ = \frac{x}{10}$$
$$10 \cdot \cos 30^\circ = \frac{x}{\cancel{10}} \cdot \cancel{10}$$
$$8.7 \doteq x$$

$$\sin 30^\circ = \frac{\text{length opposite } 30^\circ}{\text{hypotenuse}}$$
$$\sin 30^\circ = \frac{y}{10}$$
$$10 \cdot \sin 30^\circ = \frac{y}{\cancel{10}} \cdot \cancel{10}$$
$$5 = y$$

The measure of the other acute angle in the triangle is $180^\circ - 90^\circ - 30^\circ = 60^\circ$.

c. Use the Pythagorean theorem to find the length of *BC*.

$$BC^2 = 7^2 + 12^2$$
$$BC^2 = 193$$
$$BC = \sqrt{193}$$
$$BC \doteq 13.9$$

$$\tan B = \frac{\text{length opposite } \angle B}{\text{length adjacent to } \angle B}$$
$$\tan B = \frac{7}{12}$$
$$\angle B = \tan^{-1}\left(\frac{7}{12}\right)$$
$$\angle B \doteq 30.3^\circ$$

Then, $\angle C \doteq 180^\circ - 90^\circ - 30.3^\circ$

$$\angle C \doteq 59.7^\circ$$

Question 2, page 131

a. In ΔABD,

$$\tan 30^\circ = \frac{\text{length opposite } 30^\circ}{\text{length adjacent to } 30^\circ}$$

$$\tan 30^\circ = \frac{BD}{10 \text{ cm}}$$

$$10 \cdot \tan 30^\circ = \frac{BD}{\cancel{10}} \cdot \cancel{10}$$

$$5.773\ldots\text{cm} = BD$$

In ΔACD,

$$\tan 40^\circ = \frac{\text{length opposite } 40^\circ}{\text{length adjacent to } 40^\circ}$$

$$\tan 40^\circ = \frac{CD}{10 \text{ cm}}$$

$$10 \cdot \tan 40^\circ = \frac{CD}{\cancel{10}} \cdot \cancel{10}$$

$$8.390\ldots\text{cm} = CD$$

Then, $BC = BD + CD$

$BC = 5.773\ldots\text{cm} + 8.390\ldots\text{cm}$

$BC \doteq 14.2 \text{ cm}$

b. In ΔACD,

$$\tan 40^\circ = \frac{\text{length opposite } 40^\circ}{\text{length adjacent to } 40^\circ}$$

$$\tan 40^\circ = \frac{8 \text{ cm}}{CD}$$

$$CD \cdot \tan 40^\circ = \frac{8 \text{ cm}}{\cancel{CD}} \cdot \cancel{CD}$$

$$\frac{CD \cdot \cancel{\tan 40^\circ}}{\cancel{\tan 40^\circ}} = \frac{8 \text{ cm}}{\tan 40^\circ}$$

$$CD = 9.534\ldots\text{cm}$$

In ΔABD,

$$\tan 20^\circ = \frac{\text{length opposite } 2}{\text{length adjacent to } 2}$$

$$\tan 20^\circ = \frac{BD}{8 \text{ cm}}$$

$$8 \text{ cm} \cdot \tan 20^\circ = \frac{BD}{\cancel{8 \text{ cm}}} \cdot \cancel{8 \text{ cm}}$$

$$2.911\ldots\text{cm} = BD$$

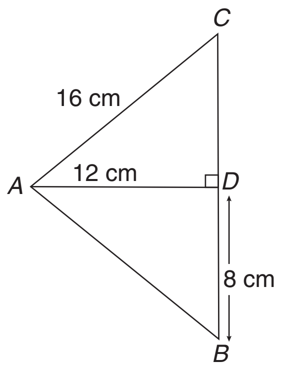
Then, $BC = BD + CD$

$BC = 2.911\ldots\text{cm} + 9.534\ldots\text{cm}$

$BC \doteq 12.4 \text{ cm}$

Question 3, page 132

Let the foot of the perpendicular be D.



In ΔACD,

$$\cos \angle CAD = \frac{\text{length adjacent } \angle CAD}{\text{hypotenuse}}$$

$$\cos \angle CAD = \frac{12 \text{ cm}}{16 \text{ cm}}$$

$$\angle CAD = \cos^{-1}\left(\frac{12 \text{ cm}}{16 \text{ cm}}\right)$$

$$\angle CAD = 41.409...^\circ$$

In ΔABD,

$$\tan \angle DAB = \frac{\text{length opposite } \angle DAB}{\text{length adjacent } \angle DAB}$$

$$\tan \angle DAB = \frac{8 \text{ cm}}{12 \text{ cm}}$$

$$\angle DAB = \tan^{-1}\left(\frac{8 \text{ cm}}{12 \text{ cm}}\right)$$

$$\angle DAB = 33.690...^\circ$$

Then, $\angle CAB = \angle CAD + \angle DAB$

$$\angle CAB = 41.409...^\circ + 33.690...^\circ$$

$$\angle CAB \doteq 75^\circ$$

Question 4, page 132

a. $\angle 1$ is the angle of depression to the ship looking from the helicopter.

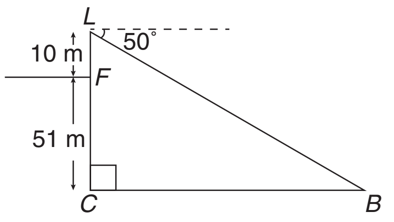
b. $\angle 2$ is the angle of elevation to the helicopter looking from the ship.

c. $\angle 3$ is the angle of depression to the submarine looking from the ship.

d. $\angle 4$ is the angle of elevation to the ship looking from the submarine.

Question 7, page 132

In the diagram, the lighthouse keeper is at L and the boat is at B. The cliff is CF.



In ΔBCL,

$$\angle BLC = 90° - 50°$$
$$\angle BLC = 40°$$
$$LC = 10 \text{ m} + 51 \text{ m}$$
$$LC = 61 \text{ m}$$
$$\text{Then, } \tan 40° = \frac{\text{length opposite } \angle 40°}{\text{length adjacent to } \angle 40°}$$
$$\tan 40° = \frac{BC}{61 \text{ m}}$$
$$61 \text{ m} \cdot \tan 40° = \frac{BC}{\cancel{61 \text{ m}}} \cdot \cancel{61 \text{ m}}$$
$$51.2 \text{ m} \doteq BC$$

The boat is not at a safe distance from the cliff because 51.2 m is less than the required 75 m.

Question 11, page 134

a. Let d represent the distance from the caller to tower 3. Then, in the right triangle on the right of the figure,

$$\sin 62° = \frac{\text{length opposite } 62°}{\text{hypotenuse}}$$
$$\sin 62° = \frac{d}{7 \text{ mi}}$$
$$7 \text{ mi} \cdot \sin 62° = \frac{d}{\cancel{7 \text{ mi}}} \cdot \cancel{7 \text{ mi}}$$
$$6.180... \text{mi} = d$$
$$6.2 \text{ mi} \doteq d$$

The distance between the caller and tower 3 is approximately 6.2 mi.

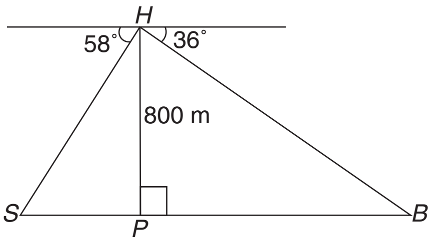
b. Let x represent the distance from tower 1 to tower 3. Then, in the right triangle on the left of the figure, use the Pythagorean theorem,

$$x^2 = (5 \text{ mi})^2 + (6.180... \text{mi})^2$$
$$x^2 = \sqrt{25 \text{ mi}^2 + (6.180... \text{mi})^2}$$
$$x = 7.949... \text{mi}$$
$$x \doteq 7.9 \text{ mi}$$

The distance between towers 1 and 3 is approximately 7.9 mi.

Question 12, page 134

a. A diagram should be drawn to explain the situation. The helicopter is at *H* and the sailboats are at *S* and *B*. The sailboat, *S*, at an angle of depression of 58º is closer to the helicopter. The steeper angle means the line of sight reaches the horizontal sooner.



b. $\angle SHP = 90^\circ - 58^\circ$
$= 32^\circ$

$$\tan \angle SHP = \frac{\text{length opposite } \angle SHP}{\text{length adjacent to } \angle SHP}$$
$$\tan 32^\circ = \frac{PS}{800 \text{ m}}$$
$$800 \text{ m} \bullet \tan 32^\circ = \frac{PS}{800 \text{ m}} \bullet \cancel{800 \text{ m}}$$
$$499.895... \text{ m} = PS$$

$\angle PHB = 90^\circ - 36^\circ$
$= 54^\circ$

$$\tan \angle PHB = \frac{\text{length opposite } \angle PHB}{\text{length adjacent to } \angle PHB}$$
$$\tan 54^\circ = \frac{PB}{800 \text{ m}}$$
$$800 \text{ m} \bullet \tan 54^\circ = \frac{PB}{800 \text{ m}} \bullet \cancel{800 \text{ m}}$$
$$1101.105... \text{ m} = PB$$

$SB = PS + PB$

$SB = 499.895...\text{m} + 1101.105...\text{m}$

$SB \doteq 1601 \text{ m}$