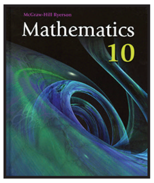
Enhance Your Understanding

Lesson 4.1: Prime Factors, GCF, and LCM

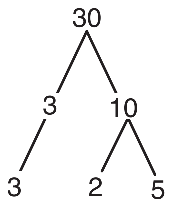
Refer to pages 215 and 222 in *Mathematics 10* for more practice.



- Page 215: 1, 2, 3, and 4
- Page 222: 13

Question 1, page 215

a. Factors of 30



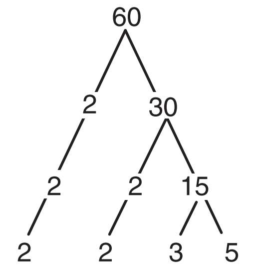
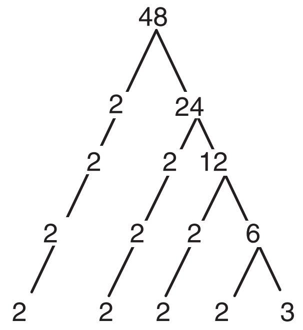
The factors cannot be further factored, so they are prime factors.

b. The number 1 cannot be written as the product of prime factors because the only whole number factors of 1 are 1 and 1, and the number 1 is not a prime number.

c. The number 0 cannot be written as the product of prime factors because the product of prime factors is non-zero.

Question 2, page 215

a.

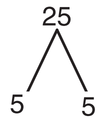
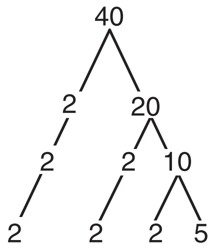
Factors of 60	Factors of 48
	

$60 = \boxed{2} \times \boxed{2} \times \boxed{3} \times 5$

$48 = \boxed{2} \times \boxed{2} \times 2 \times 2 \times \boxed{3}$

The GCF of 60 and 48 $= 2 \times 2 \times 3 = 12$.

b.

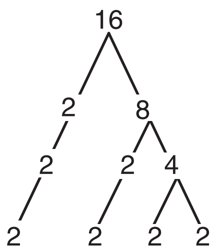
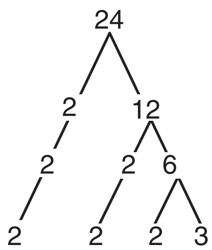
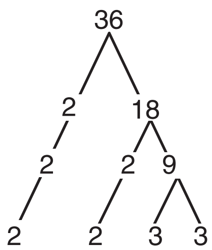
Factors of 25	Factors of 40
	

$25 = \boxed{5} \times 5$

$40 = 2 \times 2 \times 2 \times \boxed{5}$

The GCF of 25 and 40 is 5.

c.

Factors of 16	Factors of 24	Factors of 36
		

$16 = \boxed{2} \times \boxed{2} \times 2 \times 2$

$24 = \boxed{2} \times \boxed{2} \times 2 \times 3$

$36 = \boxed{2} \times \boxed{2} \times 3 \times 3$

The GCF of 16, 24, and 36 $= 2 \times 2 = 4$.

Question 3, page 215

a.

Multiples of 12	Multiples of 15
$12 \times 1 = 12$	$15 \times 1 = 15$
$12 \times 2 = 24$	$15 \times 2 = 30$
$12 \times 3 = 36$	$15 \times 3 = 45$
$12 \times 4 = 48$	$15 \times 4 = \boxed{60}$
$12 \times 5 = \boxed{60}$	

The lowest common multiple is 60.

b.

Multiples of 20	Multiples of 25
$20 \times 1 = 20$	$25 \times 1 = 25$
$20 \times 2 = 40$	$25 \times 2 = 50$
$20 \times 3 = 60$	$25 \times 3 = 75$
$20 \times 4 = 80$	$25 \times 4 = \boxed{100}$
$20 \times 5 = \boxed{100}$	$25 \times 5 = 125$
$20 \times 6 = 120$	

The lowest common multiple is 100.

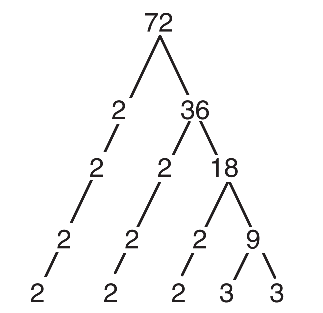
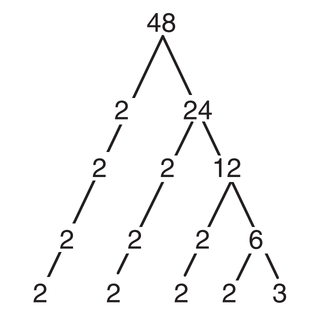
c.

Multiples of 18		Multiples of 32
$18 \times 1 = 18$	$18 \times 11 = 198$	$32 \times 1 = 32$
$18 \times 2 = 36$	$18 \times 12 = 216$	$32 \times 2 = 64$
$18 \times 3 = 54$	$18 \times 13 = 234$	$32 \times 3 = 96$
$18 \times 4 = 72$	$18 \times 14 = 252$	$32 \times 4 = 128$
$18 \times 5 = 90$	$18 \times 15 = 270$	$32 \times 5 = 160$
$18 \times 6 = 108$	$18 \times 16 = \boxed{288}$	$32 \times 6 = 192$
$18 \times 7 = 126$	$18 \times 17 = 306$	$32 \times 7 = 224$
$18 \times 8 = 144$		$32 \times 8 = 256$
$18 \times 9 = 162$		$32 \times 9 = \boxed{288}$
$18 \times 10 = 180$		$32 \times 10 = 320$

The lowest common multiple is 288.

Question 4, page 215

a.

Factors of 72	Factors of 48
	

$72 = \boxed{2} \times \boxed{2} \times \boxed{2} \times \boxed{3} \times 3$

$48 = \boxed{2} \times \boxed{2} \times \boxed{2} \times 2 \times \boxed{3}$

The GCF of 72 and 48 $= 2 \times 2 \times 2 \times 3 = 24$.

b. $72 \div 24 = 3$

$72 = 24 \times 3$

$48 \div 24 = 2$

$48 = 24 \times 2$

c. To determine the second factor, divide each of the given numbers by the GCF, 24.

Question 13, page 222

Find the greatest common factor of 30, 48, and 36.

Factors of 30: 1, 2, 3, 5, **6**, 10, 15, 30

Factors of 48: 1, 2, 3, 4, **6**, 8, 12, 16, 24, 48

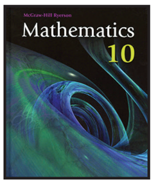
Factors of 36: 1, 2, 3, 4, **6**, 9, 12, 18, 36

The GCF is 6; Nikolai needs to divide the items among six containers.

In each of the 6 containers, there will be 5 pencils, 8 pens, and 6 erasers.

Lesson 4.2: Mixed and Entire Radicals

Refer to pages 158 and 192 in *Mathematics 10* for more practice.



- Page 158: 1, 3a, 3b, 3c, 3d, 3e, 4a, 4b, 4c, 4d, 4e, 5, 6, and 7
- Page 192: 4, 5a, 5b, 5c, 6a, 6b, 6c, and 7a, b, c, d

Question 1, page 158

a. $$\begin{aligned} 7^2 &= (7)(7) \\ &= 49 \end{aligned}$$

b. $$\begin{aligned} -50^2 &= -(50)(50) \\ &= -2\,500 \end{aligned}$$

c. $$\begin{aligned} (-3)^2 &= (-3)(-3) \\ &= 9 \end{aligned}$$

d. $$\begin{aligned} \frac{4^2}{5} &= \frac{(4)(4)}{5} \\ &= \frac{16}{5} \end{aligned}$$

e. $$\begin{aligned} \frac{3}{2^2} &= \frac{3}{(2)(2)} \\ &= \frac{3}{4} \end{aligned}$$

f. $$\begin{aligned} \left(\frac{3}{4}\right)^2 &= \left(\frac{3}{4}\right)\left(\frac{3}{4}\right) \\ &= \frac{9}{16} \end{aligned}$$

Question 3, page 158

a. $$\begin{aligned} \sqrt{49} &= \sqrt{7 \times 7} \\ &= \sqrt{7^2} \\ &= 7 \end{aligned}$$

b. $\sqrt{169} = \sqrt{13 \times 13}$
$= \sqrt{13^2}$
$= 13$

c. $\sqrt{(25)(4)} = \sqrt{(5 \times 5)(2 \times 2)}$
$= \sqrt{5^2 \times 2^2}$
$= \sqrt{5^2} \times \sqrt{2^2}$
$= 5 \times 2$
$= 10$

d. $\frac{16}{\sqrt{64}} = \frac{16}{\sqrt{8 \times 8}}$
$= \frac{16}{\sqrt{8^2}}$
$= \frac{16}{8}$
$= 2$

e. $\frac{\sqrt{36}}{3} = \frac{\sqrt{6 \times 6}}{3}$
$= \frac{\sqrt{6^2}}{3}$
$= \frac{6}{3}$
$= 2$

Question 4, page 158

a. $\sqrt[3]{1} = \sqrt[3]{1 \times 1 \times 1}$
$= \sqrt[3]{1^3}$
$= 1$

b. $\sqrt[3]{(8)(27)} = \sqrt[3]{(2 \times 2 \times 2)(3 \times 3 \times 3)}$
$= \sqrt[3]{2^3 \times 3^3}$
$= \sqrt[3]{2^3} \times \sqrt[3]{3^3}$
$= 2 \times 3$
$= 6$

c. $\sqrt[3]{8\,000} = \sqrt[3]{20 \times 20 \times 20}$
$= \sqrt[3]{20^3}$
$= 20$

d. $\dfrac{\sqrt[3]{64}}{2} = \dfrac{\sqrt[3]{4 \times 4 \times 4}}{2}$

$= \dfrac{\sqrt[3]{4^3}}{2}$

$= \dfrac{4}{2}$

$= 2$

e. $\sqrt[3]{\dfrac{27}{125}} = \dfrac{\sqrt[3]{3 \times 3 \times 3}}{\sqrt[3]{5 \times 5 \times 5}}$

$= \dfrac{\sqrt[3]{3^3}}{\sqrt[3]{5^3}}$

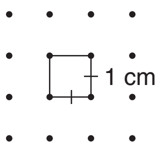
$= \dfrac{3}{5}$

Question 5, page 158

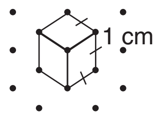
a. The number 1 is a perfect square and a perfect cube.

$1 \times 1 = 1^2 = 1$

$1 \times 1 \times 1 = 1^3 = 1$

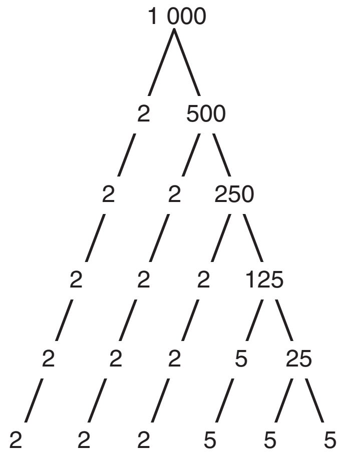


Area = 1 cm^2



Volume = 1 cm^3

b. The number 1000 is a perfect cube.

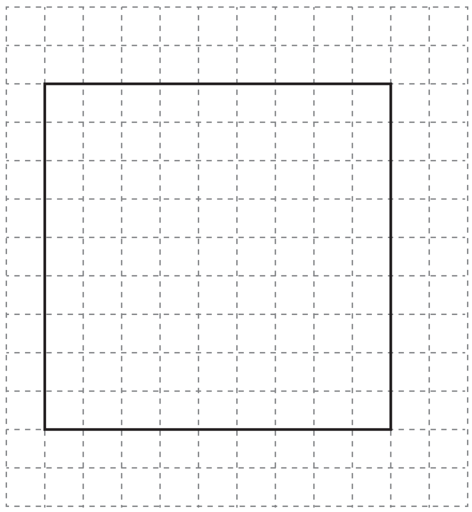


$(2 \times 5)(2 \times 5)(2 \times 5) = 10 \times 10 \times 10$

$10 \times 10 \times 10 = 10^3 = 1\,000$

So, $\sqrt[3]{1\,000} = 10$

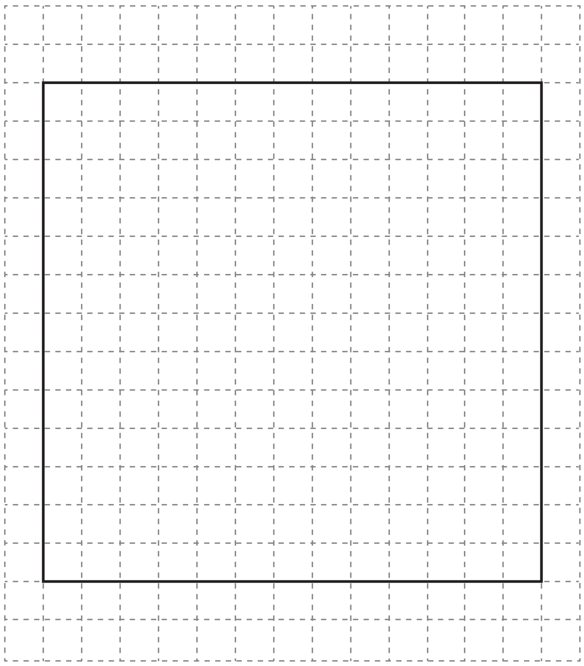
c. The number 81 is a perfect square.



A square with side length 9 units has an area of 81 units².

$$\begin{aligned} A &= s^2 \\ &= 9 \text{ units} \times 9 \text{ units} \\ &= 81 \text{ units}^2 \end{aligned}$$

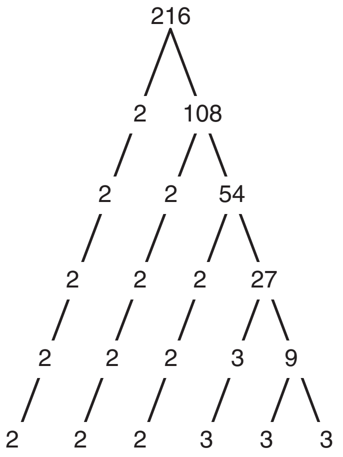
d. The number 169 is a perfect square.



A square with side length 13 units has an area of 169 units^2.

$$\begin{aligned} A &= s^2 \\ &= 13 \text{ units} \times 13 \text{ units} \\ &= 169 \text{ units}^2 \end{aligned}$$

e. The number 216 is a perfect cube.

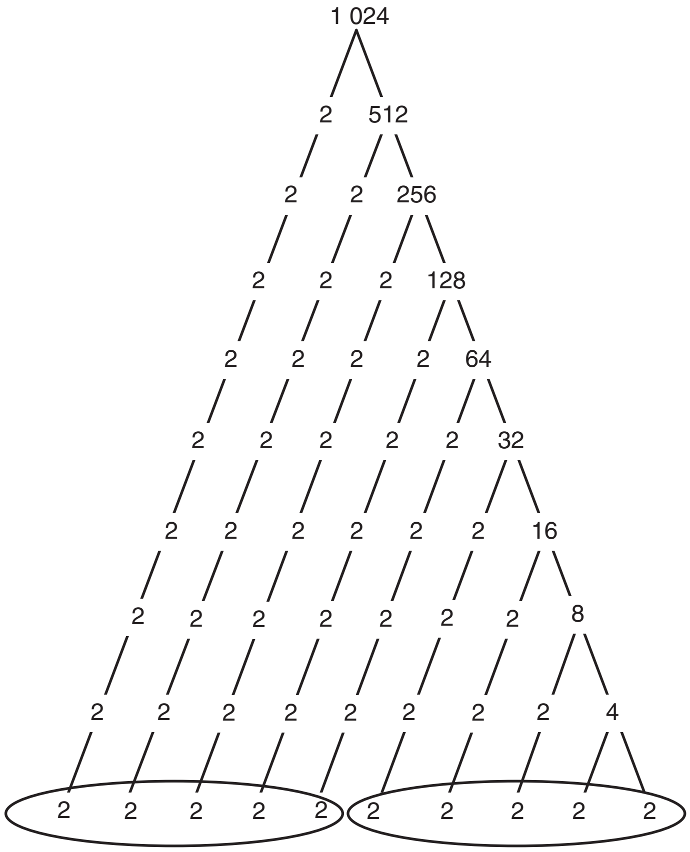


$$(2 \times 3)(2 \times 3)(2 \times 3) = 6 \times 6 \times 6$$

$$6 \times 6 \times 6 = 6^3 = 216$$

$$\text{So, } \sqrt[3]{216} = 6$$

f. The 1024 is a perfect square.



$$(2 \times 2 \times 2 \times 2 \times 2)(2 \times 2 \times 2 \times 2 \times 2) = 32 \times 32$$

$$32 \times 32 = 32^2 = 1\,024$$

$$\text{So, } \sqrt{1\,024} = 32$$

Question 6, page 158

a. The number 144 is a perfect square because $144 = 12^2$.

b. The number 2197 is a perfect cube because $2\,197 = 13^3$.

c. The number 16 is a perfect square because $16 = 4^2$.

d. The number 225 is a perfect square because $225 = 15^2$.

e. The number 15 625 is a perfect square and a perfect cube.

$15\,625 = 125^2$

$15\,625 = 25^3$

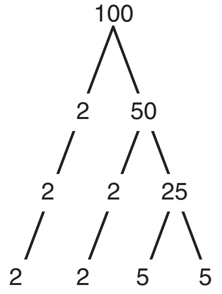
f. The number 117 649 is a perfect square and a perfect cube.

$117\,649 = 343^2$

$117\,649 = 49^3$

Question 7, page 158

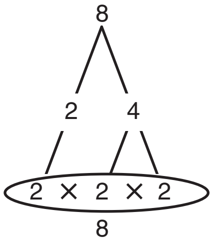
a. Write 100 as the product of its prime factors using a factor tree. Group the prime factors into two equal groups. Then, identify the factor that can be squared to form 100.



$$(2 \times 5)(2 \times 5) = 100$$
$$10 \times 10 = 100$$
$$10^2 = 100$$

Therefore, $\sqrt{100} = 10$.

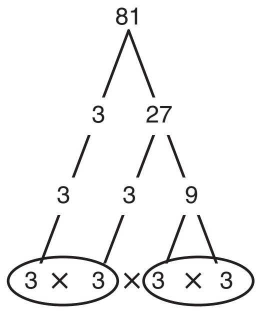
b. Write 8 as the product of its prime factors using a factor tree. Then, identify the factor that can be cubed to form 8.



$$2^3 = 8$$

Therefore, $\sqrt[3]{8} = 2$.

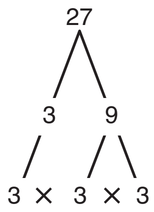
c. Write 81 as the product of its prime factors using a factor tree. Group the prime factors into two equal groups. Then, identify the factor that can be squared to form 81.



$$(3 \times 3)(3 \times 3) = 81$$
$$9 \times 9 = 81$$
$$9^2 = 81$$

Therefore, $\sqrt{81} = 9$.

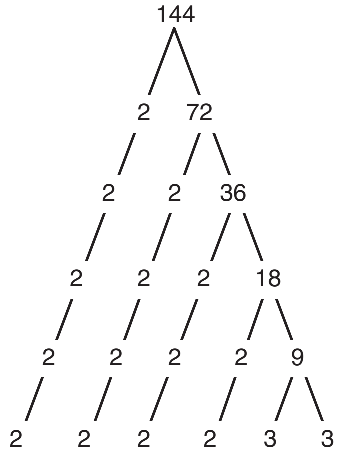
d. Write 27 as the product of its prime factors using a factor tree. Then, identify the factor that can be cubed to form 27.



$$3^3 = 27$$

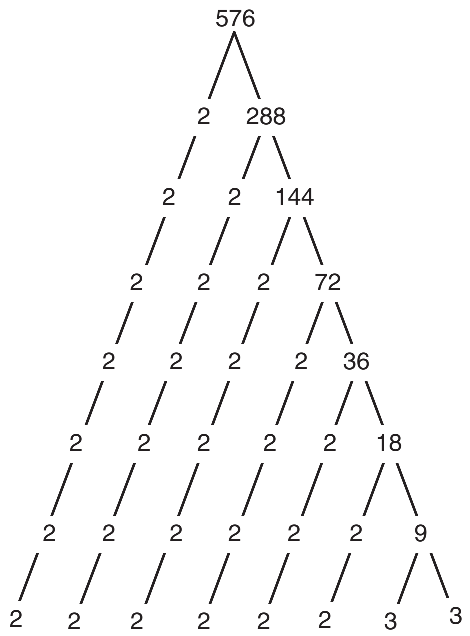
Therefore, $\sqrt[3]{27} = 3$.

e. Write 144 as the product of its prime factors using a factor tree. Group the prime factors into two equal groups. Then, identify the factors that can be squared to form 144.



$$(2 \times 2 \times 3)(2 \times 2 \times 3) = 144$$
$$12 \times 12 = 144$$
$$12^2 = 144$$
Therefore, $\sqrt{144} = 12$.

f. Write 576 as the product of its prime factors using a factor tree. Group the prime factors into two equal groups. Then, identify the factor that can be squared to form 576.



$$(2 \times 2 \times 2 \times 3)(2 \times 2 \times 2 \times 3) = 576$$
$$24 \times 24 = 576$$
$$24^2 = 576$$
Therefore, $\sqrt{576} = 24$.

Question 4, page 192

a. $$\begin{aligned} 3\sqrt{11} &= \sqrt{3^2} \times \sqrt{11} \\ &= \sqrt{3^2 \times 11} \\ &= \sqrt{9 \times 11} \\ &= \sqrt{99} \end{aligned}$$

b. $$\begin{aligned} 7\sqrt{2} &= \sqrt{7^2} \times \sqrt{2} \\ &= \sqrt{7^2 \times 2} \\ &= \sqrt{49 \times 2} \\ &= \sqrt{98} \end{aligned}$$

c. $$\begin{aligned} 3\sqrt{5} &= \sqrt{3^2} \times \sqrt{5} \\ &= \sqrt{3^2 \times 5} \\ &= \sqrt{9 \times 5} \\ &= \sqrt{45} \end{aligned}$$

d. $$\begin{aligned} 2\sqrt{7} &= \sqrt{2^2} \times \sqrt{7} \\ &= \sqrt{2^2 \times 7} \\ &= \sqrt{4 \times 7} \\ &= \sqrt{28} \end{aligned}$$

e. $$\begin{aligned} 3\sqrt{3} &= \sqrt{3^2} \times \sqrt{3} \\ &= \sqrt{3^2 \times 3} \\ &= \sqrt{9 \times 3} \\ &= \sqrt{27} \end{aligned}$$

f. $$\begin{aligned} 10\sqrt{6} &= \sqrt{10^2} \times \sqrt{6} \\ &= \sqrt{10^2 \times 6} \\ &= \sqrt{100 \times 6} \\ &= \sqrt{600} \end{aligned}$$

Question 5, page 192

a. $$\begin{aligned}2\sqrt[3]{7} &= \sqrt[3]{2^3} \times \sqrt[3]{7}\\ &= \sqrt[3]{2^3 \times 7}\\ &= \sqrt[3]{8 \times 7}\\ &= \sqrt[3]{56}\end{aligned}$$

b. $$\begin{aligned}3\sqrt[3]{3} &= \sqrt[3]{3^3} \times \sqrt[3]{3}\\ &= \sqrt[3]{3^3 \times 3}\\ &= \sqrt[3]{27 \times 3}\\ &= \sqrt[3]{81}\end{aligned}$$

c. $$\begin{aligned}10\sqrt[3]{5} &= \sqrt[3]{10^3} \times \sqrt[3]{5}\\ &= \sqrt[3]{10^3 \times 5}\\ &= \sqrt[3]{1\,000 \times 5}\\ &= \sqrt[3]{5\,000}\end{aligned}$$

Question 6, page 193

a. $$\begin{aligned}\sqrt{12} &= \sqrt{4 \times 3}\\ &= \sqrt{2^2 \times 3}\\ &= \sqrt{2^2} \times \sqrt{3}\\ &= 2\sqrt{3}\end{aligned}$$

b. $$\begin{aligned}\sqrt{50} &= \sqrt{25 \times 2}\\ &= \sqrt{5^2 \times 2}\\ &= \sqrt{5^2} \times \sqrt{2}\\ &= 5\sqrt{2}\end{aligned}$$

c. $$\begin{aligned}\sqrt{48} &= \sqrt{16 \times 3}\\ &= \sqrt{4^2 \times 3}\\ &= \sqrt{4^2} \times \sqrt{3}\\ &= 4\sqrt{3}\end{aligned}$$

Question 7, page 193

a. $\sqrt[3]{24} = \sqrt[3]{8 \times 3}$
$= \sqrt[3]{8} \times \sqrt[3]{3}$
$= \sqrt[3]{2^3} \times \sqrt[3]{3}$
$= 2\sqrt[3]{3}$

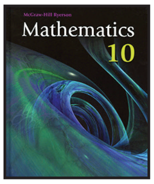
b. $\sqrt[3]{54} = \sqrt[3]{27 \times 2}$
$= \sqrt[3]{27} \times \sqrt[3]{2}$
$= \sqrt[3]{3^3} \times \sqrt[3]{2}$
$= 3\sqrt[3]{2}$

c. $\sqrt[3]{243} = \sqrt[3]{27 \times 9}$
$= \sqrt[3]{27} \times \sqrt[3]{9}$
$= \sqrt[3]{3^3} \times \sqrt[3]{9}$
$= 3\sqrt[3]{9}$

d. $\sqrt[3]{40} = \sqrt[3]{8 \times 5}$
$= \sqrt[3]{8} \times \sqrt[3]{5}$
$= \sqrt[3]{2^3} \times \sqrt[3]{5}$
$= 2\sqrt[3]{5}$

Lesson 4.3: The Irrational Number System

Refer to pages 193 and 198 in *Mathematics 10* for more practice.



- Page 193: 8a, 9
- Page 198: 22b, 23

Question 8, page 193

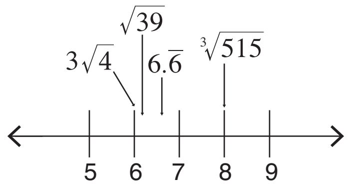
a. $\frac{5}{8} = 0.625, 0.\overline{6} = 0.666..., \sqrt{0.25} = 0.5, \sqrt[3]{0.84} = 0.943...$

The numbers in order from least to greatest are $\sqrt{0.25}, \frac{5}{8}, 0.\overline{6}, \sqrt[3]{0.84}$.

The irrational number is $\sqrt[3]{0.84}$.

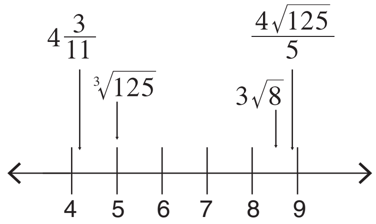
Question 9, page 193

a. $3\sqrt{4} = 6, 6.\overline{6} = 6.666..., \sqrt{39} = 6.244..., \sqrt[3]{515} = 8.015...$



The irrational numbers are $\sqrt{39}$ and $\sqrt[3]{515}$.

b. $4\frac{3}{11} = 4.\overline{27}, \sqrt[3]{125} = 5, \frac{4\sqrt{125}}{5} \doteq 8.944..., 3\sqrt{8} = 8.485...$



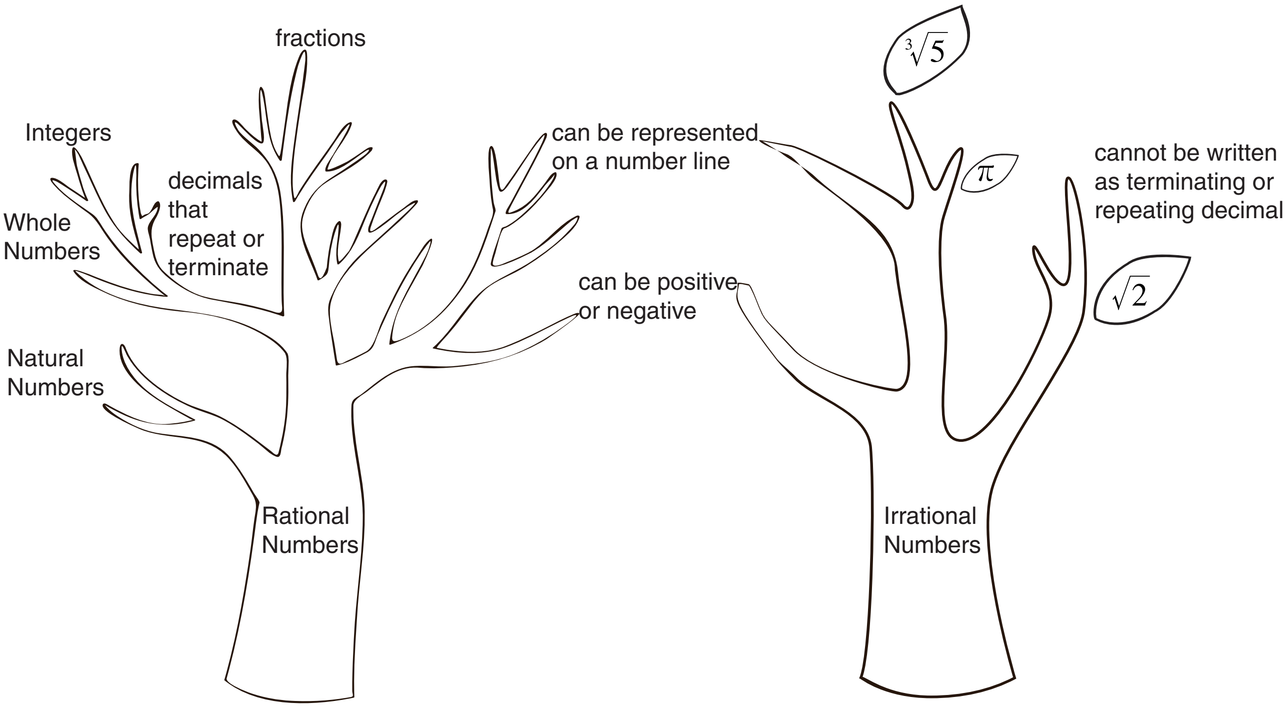
The irrational numbers are $\frac{4\sqrt{125}}{5}$ and $3\sqrt{8}$.

Question 22b, page 198

b. $18^{\frac{1}{2}} = \sqrt{18} = 4.242...$, $\sqrt{36} = 6$, $6.\overline{2} = 6.222...$, and $2\sqrt[3]{27} = 6$

The only irrational number in this set is $18^{\frac{1}{2}}$. In order from greatest to least, they are $6.\overline{2}$, $\sqrt{36}$, $2\sqrt[3]{27}$, and $18^{\frac{1}{2}}$. Because $\sqrt{36} = 2\sqrt[3]{27} = 6$ these two could be written in either order.

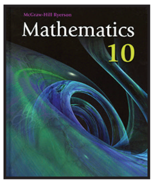
Question 23, page 198



The idea tree diagram has one tree with branches that describe Rational Numbers and one tree with branches that describe Irrational Numbers. The two inside branches on each tree connect similarities between Rational and Irrational Numbers.

Lesson 4.4: Exponent Laws

Refer to pages 169, 180, and 192 in *Mathematics 10* for more practice.



- Page 169: 2, 3, 4, 5, 6, 7, 8, and 21
- Page 180: 1, 2, 4, 5, 7, and 11
- Page 192: 1, 2, 3, 11, 12, 15, 16, 19, and 20

Question 2, page 169

a. $b^{-3} = \frac{1}{b^3}$

b. $$\begin{aligned} xy^{-4} &= x\left(\frac{1}{y^4}\right) \\ &= \frac{x}{y^4} \end{aligned}$$

c. $$\begin{aligned} 2x^{-2} &= 2\left(\frac{1}{x^2}\right) \\ &= \frac{2}{x^2} \end{aligned}$$

d. $$\begin{aligned} 2x^2y^{-1} &= 2x^2\left(\frac{1}{y^1}\right) \\ &= \frac{2x^2}{y} \end{aligned}$$

e. $$\begin{aligned} -4x^{-5} &= -4\left(\frac{1}{x^5}\right) \\ &= -\frac{4}{x^5} \end{aligned}$$

f. $$\begin{aligned} -2x^{-3}y^{-4} &= -2\left(\frac{1}{x^3}\right)\left(\frac{1}{y^4}\right) \\ &= -\frac{2}{x^3y^4} \end{aligned}$$

Question 3, page 169

Daniel's answer is correct because the negative exponent affects the x variable only.

$$\begin{aligned}\frac{2x^{-3}}{y^5} &= \left(\frac{2}{y^5}\right)\left(\frac{1}{x^3}\right)\\ &= \frac{2}{x^3y^5}\end{aligned}$$

Question 4, page 169

a. $$\begin{aligned}(4^3)(4^{-5}) &= 4^{3+(-5)}\\ &= 4^{-2}\\ &= \frac{1}{4^2}\end{aligned}$$

b. $$\begin{aligned}\frac{3^{-4}}{3^{-2}} &= 3^{-4-(-2)}\\ &= 3^{-4+2}\\ &= 3^{-2}\\ &= \frac{1}{3^2}\end{aligned}$$

c. $$\begin{aligned}\frac{12^3}{12^7} &= 12^{3-7}\\ &= 12^{-4}\\ &= \frac{1}{12^4}\end{aligned}$$

d. $$\begin{aligned}\left(\frac{8^{-1}}{8^0}\right)^3 &= (8^{(-1-0)})^3\\ &= 8^{(-1\cdot 3)}\\ &= 8^{-3}\\ &= \frac{1}{8^3}\end{aligned}$$

e. $$\begin{aligned}(5^4)^{-2} &= 5^{4\cdot(-2)}\\ &= 5^{-8}\\ &= \frac{1}{5^8}\end{aligned}$$

f. $$\begin{aligned}[(3^2)(2^{-5})]^3 &= (3^2)^3(2^{-5})^3 \\ &= 3^{(2\cdot3)}\cdot 2^{(-5\cdot3)} \\ &= 3^6\cdot 2^{-15} \\ &= 3^6\left(\frac{1}{2^{15}}\right) \\ &= \frac{3^6}{2^{15}}\end{aligned}$$

g. $$\begin{aligned}\left(\frac{5^2}{4^2}\right)^{-1} &= \frac{5^{2\cdot(-1)}}{4^{2\cdot(-1)}} \\ &= \frac{5^{-2}}{4^{-2}} \\ &= \frac{\frac{1}{5^2}}{\frac{1}{4^2}} \\ &= \frac{1}{5^2}\div\frac{1}{4^2} \\ &= \frac{1}{5^2}\cdot\frac{4^2}{1} \\ &= \frac{4^2}{5^2}\end{aligned}$$

h. $$\begin{aligned}(3.2^{-2})^{-3} &= 3.2^{-2\cdot(-3)} \\ &= 3.2^6\end{aligned}$$

i. $$\begin{aligned}4[(2)^{-1}(2)^{-2}]^{-1} &= 4(2^{-1\cdot(-1)})(2^{-2\cdot(-1)}) \\ &= 4(2^1)(2^2) \\ &= 4(2^{1+2}) \\ &= 4(2^3)\end{aligned}$$

Question 5, page 169

a. $$\begin{aligned}\frac{1}{s^2t^{-6}} &= \left(\frac{1}{s^2}\right)\left(\frac{1}{t^{-6}}\right) \\ &= \left(\frac{1}{s^2}\right)(t^6) \\ &= \frac{t^6}{s^2}\end{aligned}$$

b. $$\begin{aligned}[(h)^7(h)^{-2}]^{-2} &= [(h)^{7+(-2)}]^{-2} \\ &= (h^5)^{-2} \\ &= h^{5\cdot(-2)} \\ &= h^{-10} \\ &= \frac{1}{h^{10}}\end{aligned}$$

c. $$\begin{aligned}\frac{8t}{t^{-3}} &= \frac{8t^1}{t^{-3}} \\ &= 8 \bullet t^{1-(-3)} \\ &= 8 \bullet t^{1+3} \\ &= 8t^4\end{aligned}$$

d. $$\begin{aligned}(2x^{-4})^3 &= 2^3 \bullet x^{(-4\cdot 3)} \\ &= 8 \bullet x^{-12} \\ &= 8\left(\frac{1}{x^{12}}\right) \\ &= \frac{8}{x^{12}}\end{aligned}$$

e. $$\begin{aligned}\left(\frac{n^4}{n^{-4}}\right)^{-3} &= (n^{(4-(-4))})^{-3} \\ &= (n^8)^{-3} \\ &= n^{8\cdot(-3)} \\ &= n^{-24} \\ &= \frac{1}{n^{24}}\end{aligned}$$

f. $$\begin{aligned}[(xy^4)^{-3}]^{-2} &= (xy^4)^{(-3\cdot(-2))} \\ &= (xy^4)^6 \\ &= x^{(1\cdot 6)}y^{(4\cdot 6)} \\ &= x^6y^{24}\end{aligned}$$

Question 6, page 169

a. $$\begin{aligned}(0.5^2)^{-3} &= 0.5^{2\cdot(-3)}\\ &= 0.5^{-6}\\ &= \frac{1}{0.5^6}\\ &= 64\end{aligned}$$

b. $$\begin{aligned}\left[\left(\frac{2}{3}\right)^3\right]^{-3} &= \left(\frac{2}{3}\right)^{(3\cdot(-3))}\\ &= \left(\frac{2}{3}\right)^{(-9)}\\ &= \frac{2^{-9}}{3^{-9}}\\ &= \frac{\frac{1}{2^9}}{\frac{1}{3^9}}\\ &= \frac{1}{2^9} \div \frac{1}{3^9}\\ &= \frac{1}{2^9} \bullet \frac{3^9}{1}\\ &= \frac{3^9}{2^9}\\ &\doteq 38.4434\end{aligned}$$

c. $$\begin{aligned}[(5)(5^3)]^{-1} &= (5^{(1+3)})^{-1}\\ &= (5^4)^{-1}\\ &= 5^{4\cdot(-1)}\\ &= 5^{-4}\\ &= \frac{1}{5^4}\\ &= 0.0016\end{aligned}$$

d. $$\begin{aligned}\left(\frac{6^4}{6^4}\right)^{-3} &= (1)^{-3}\\ &= \frac{1}{1^3}\\ &= 1\end{aligned}$$

e. $\left(\frac{8}{8^3}\right)^{-4} = (8^{(1-3)})^{-4}$
$= (8^{-2})^{-4}$
$= 8^{-2\cdot(-4)}$
$= 8^8$
$= 16\,777\,216$

f. $\left[\left(\frac{3}{4}\right)^{-4} \div \left(\frac{3}{4}\right)^{2}\right]^{-1} = \left[\left(\frac{3}{4}\right)^{(-4-2)}\right]^{-1}$
$= \left[\left(\frac{3}{4}\right)^{-6}\right]^{-1}$
$= \left(\frac{3}{4}\right)^{-6\cdot(-1)}$
$= \left(\frac{3}{4}\right)^{6}$
$= \frac{3^6}{4^6}$
$\doteq 0.1780$

Question 7, page 170

a. The population of P is 20 000 beetles at time $n = 0$. Use the given formula. For four years ago, substitute $n = –4$.

$P = 20\,000(2)^n$
$P = 20\,000(2)^{-4}$
$P = \frac{20\,000}{2^4}$
$P = 1\,250$

Four years ago, the beetle population was 1 250.

For eight years ago, substitute $n = –8$.

$P = 20\,000(2)^n$
$P = 20\,000(2)^{-8}$
$P = \frac{20\,000}{2^8}$
$P = 78.125$

Eight years ago, the beetle population was approximately 78.

b. For two years from now, substitute $n = 2$.

$$P = 20\,000(2)^n$$
$$P = 20\,000(2)^2$$
$$P = 20\,000(4)$$
$$P = 80\,000$$

In two years, the population will be 80 000 if conditions remain ideal.

Question 8, page 170

In 1996, $n = 0$. In 2010, $n = 14$. Substitute $n = 14$ into the given formula.

$$S = 300\,000(1.05)^n$$
$$S = 300\,000(1.05)^{14}$$
$$S = 593\,979.479...$$

The projected sales for 2010 would be approximately $593 979.48.

Question 21, page 172

$$\begin{aligned}\left(\frac{2^5}{8^2}\right)^{-2} &= \frac{2^{5\cdot(-2)}}{8^{2\cdot(-2)}}\\ &= \frac{2^{-10}}{8^{-4}}\\ &= \frac{2^{-10}}{(2^3)^{-4}}\\ &= \frac{2^{-10}}{2^{3\cdot(-4)}}\\ &= \frac{2^{-10}}{2^{-12}}\\ &= 2^{-10-(-12)}\\ &= 2^2\\ &= 4\end{aligned}$$

Question 1, page 180

a. $(x^3)(x^{\frac{7}{3}}) = x^{\left(3+\frac{7}{3}\right)}$
$= x^{\left(\frac{3}{1}+\frac{7}{3}\right)}$
$= x^{\left(\frac{9}{3}+\frac{7}{3}\right)}$
$= x^{\frac{16}{3}}$

b. $(b^{\frac{1}{5}})(b^{\frac{9}{5}}) = b^{\left(\frac{1}{5}+\frac{9}{5}\right)}$
$= b^{\frac{10}{5}}$
$= b^2$

c. $(a^2)^{\frac{3}{2}} = a^{\left(2\cdot\frac{3}{2}\right)}$
$= a^{\left(\frac{2}{1}\cdot\frac{3}{2}\right)}$
$= a^{\frac{6}{2}}$
$= a^3$

d. $(k^{4.8})(k^3) = k^{(4.8+3)}$
$= k^{7.8}$

e. $(16)^{0.25} = (16)^{\frac{1}{4}}$
$= (2^4)^{\frac{1}{4}}$
$= 2^{\left(4\cdot\frac{1}{4}\right)}$
$= 2^{\left(\frac{4}{1}\cdot\frac{1}{4}\right)}$
$= 2^{\frac{4}{4}}$
$= 2^1$
$= 2$

or

$(16)^{0.25} = (16)^{\frac{1}{4}}$
$= \sqrt[4]{16}$
$= \sqrt[4]{2^4}$
$= 2$

f. $$\left(\frac{-8a^6}{27}\right)^{\frac{1}{3}} = \frac{(-8a^6)^{\frac{1}{3}}}{27^{\frac{1}{3}}}$$
$$= \frac{(-8)^{\frac{1}{3}} \bullet a^{\left(6 \cdot \frac{1}{3}\right)}}{27^{\frac{1}{3}}}$$
$$= \frac{\sqrt[3]{-8} \bullet a^{\frac{6}{3}}}{\sqrt[3]{27}}$$
$$= \frac{\sqrt[3]{(-2)^3} \bullet a^2}{\sqrt[3]{3^3}}$$
$$= \frac{-2a^2}{3}$$

g. $$\left(2x^{\frac{1}{3}}\right)\left(-4x^{\frac{5}{3}}\right) = (2 \bullet (-4))x^{\left(\frac{1}{3}+\frac{5}{3}\right)}$$
$$= -8x^{\frac{1}{3}+\frac{5}{3}}$$
$$= -8x^{\frac{6}{3}}$$
$$= -8x^2$$

h. $$(9x^2)^{\frac{3}{2}} = (9)^{\frac{3}{2}} \bullet (x^2)^{\frac{3}{2}}$$
$$= \left(\sqrt{9}\right)^3 \bullet \left(x^{2 \cdot \frac{3}{2}}\right)$$
$$= 3^3 \bullet x^3$$
$$= 27x^3$$

i. $$(25x^2)^{0.5} = (25x^2)^{\frac{1}{2}}$$
$$= 25^{\frac{1}{2}} \bullet x^{\left(2 \cdot \frac{1}{2}\right)}$$
$$= \sqrt{25} \bullet x^1$$
$$= \sqrt{5^2} \bullet x$$
$$= 5x$$

Question 2, page 180

a. $$(x^3)\left(x^{\frac{-2}{3}}\right) = x^{3+\frac{-2}{3}}$$
$$= x^{\frac{3}{1}+\frac{-2}{3}}$$
$$= x^{\frac{9}{3}+\frac{-2}{3}}$$
$$= x^{\frac{7}{3}}$$

b. $(81^{-0.25})^3 = (81^{-\frac{1}{4}})^3$
$= 81^{-\frac{1}{4}\cdot 3}$
$= 81^{-\frac{3}{4}}$
$= (\sqrt[4]{81})^{-3}$
$= (\sqrt[4]{3^4})^{-3}$
$= (3)^{-3}$
$= \frac{1}{3^3}$

c. $\frac{(m^{-2})^{\frac{2}{3}}}{(m^{\frac{1}{2}})^4} = \frac{m^{\left(-2\cdot\frac{2}{3}\right)}}{m^{\left(\frac{1}{2}\cdot 4\right)}}$
$= \frac{m^{\left(-\frac{4}{3}\right)}}{m^{\left(\frac{4}{2}\right)}}$
$= \frac{m^{\left(-\frac{4}{3}\right)}}{m^2}$
$= m^{-\frac{4}{3}-\frac{2}{1}}$
$= m^{-\frac{4}{3}-\frac{6}{3}}$
$= m^{-\frac{10}{3}}$
$= \frac{1}{m^{\frac{10}{3}}}$

d. $(9p^2)^{-\frac{1}{2}}\left(p^{-\frac{3}{2}}\right) = (9^{-\frac{1}{2}})\left(p^{2\cdot\left(-\frac{1}{2}\right)}\right)\left(p^{-\frac{3}{2}}\right)$
$= \frac{1}{\sqrt{9}} \cdot p^{\left(-\frac{2}{2}\right)+\left(-\frac{3}{2}\right)}$
$= \frac{1}{\sqrt{3^2}}\left(p^{-\frac{5}{2}}\right)$
$= \frac{1}{3}\left(\frac{1}{p^{\frac{5}{2}}}\right)$
$= \frac{1}{3p^{\frac{5}{2}}}$

e. $$\left[\frac{x^{-2}}{(xy)^4}\right]^{1.5} = \frac{x^{(-2)(1.5)}}{(xy)^{(4)(1.5)}}$$
$$= \frac{x^{-3}}{(xy)^6}$$
$$= \frac{x^{-3}}{x^6y^6}$$
$$= \frac{1}{x^3x^6y^6}$$
$$= \frac{1}{x^{3+6}y^6}$$
$$= \frac{1}{x^9y^6}$$

f. $$\left[\frac{4x^{-2}}{9y^{-4}}\right]^{-\frac{5}{2}} = \frac{4^{-\frac{5}{2}} \bullet x^{-2\cdot\left(-\frac{5}{2}\right)}}{9^{-\frac{5}{2}} \bullet y^{-4\cdot\left(-\frac{5}{2}\right)}}$$
$$= \frac{(\sqrt{4})^{-5} \bullet x^{\frac{10}{2}}}{(\sqrt{9})^{-5} \bullet y^{\frac{20}{2}}}$$
$$= \frac{(\sqrt{2^2})^{-5} \bullet x^5}{(\sqrt{3^3})^{-5} \bullet y^{10}}$$
$$= \frac{(2)^{-5} \bullet x^5}{(3)^{-5} \bullet y^{10}}$$
$$= \frac{3^5 \bullet x^5}{2^5 \bullet y^{10}}$$
$$= \frac{3^5x^5}{2^5y^{10}}$$

Question 4, page 180

a. $$8^{\frac{2}{3}} = (\sqrt[3]{8})^2$$
$$= (\sqrt[3]{2^3})^2$$
$$= 2^2$$
$$= 4$$

b. $$16^{\frac{1}{4}} = \sqrt[4]{16}$$
$$= \sqrt[4]{2^4}$$
$$= 2$$

c. $$-27^{\frac{4}{3}} = -(\sqrt[3]{27})^4$$
$$= -(\sqrt[3]{3^3})^4$$
$$= -(3)^4$$
$$= -81$$

d. $(3^{\frac{1}{6}})(3^{\frac{5}{6}}) = 3^{\frac{1}{6}+\frac{5}{6}}$
$= 3^{\frac{6}{6}}$
$= 3^1$
$= 3$

e. $\left(\frac{36x^0}{25}\right)^{1.5} = \left(\frac{36x^0}{25}\right)^{\frac{3}{2}}$
$= \frac{36^{\frac{3}{2}} \bullet x^{0 \cdot \frac{3}{2}}}{25^{\frac{3}{2}}}$
$= \frac{(\sqrt{36})^3 \bullet x^0}{(\sqrt{25})^3}$
$= \frac{(\sqrt{6^2})^3 \bullet 1}{(\sqrt{5^2})^3}$
$= \frac{6^3}{5^3}$
$= \frac{216}{125}$

f. $\frac{6^{-2}}{36^{-\frac{1}{2}}} = \frac{6^{-2}}{(\sqrt{36})^{-1}}$
$= \frac{6^{-2}}{(\sqrt{6^2})^{-1}}$
$= \frac{6^{-2}}{6^{-1}}$
$= 6^{-2-(-1)}$
$= 6^{-2+1}$
$= 6^{-1}$
$= \frac{1}{6}$

Question 5, page 180

a. $(81^{-0.25})^3 = 81^{(-0.25)(3)}$
$= 81^{-0.75}$
$= \frac{1}{81^{0.75}}$
$\doteq 0.0370$

b. $(8^3)(8^{1.2}) = 8^{3+1.2}$
$= 8^{4.2}$
$\doteq 6\,208.3751$

c. $\left(\frac{2^5}{5^2}\right)^{-\frac{3}{2}} = \left(\frac{32}{25}\right)^{-1.5}$

$= \left(\frac{25}{32}\right)^{1.5}$

$\doteq 0.6905$

d. $\left(\frac{2^3}{8^2}\right)^{\frac{2}{3}} = \left(\frac{8}{64}\right)^{\frac{2}{3}}$

$= 0.25$

e. $\left(\frac{-64}{6^{\frac{1}{2}}}\right)^{\frac{4}{3}} = \frac{(-64)^{\frac{4}{3}}}{6^{\frac{1}{2}\cdot\frac{4}{3}}}$

$= \frac{(-64)^{\frac{4}{3}}}{6^{\frac{4}{6}}}$

$\doteq 77.5305$

f. $\frac{\left(2^{\frac{1}{2}}\right)^3}{16} = \frac{2^{\frac{1}{2}\cdot 3}}{16}$

$= \frac{2^{\frac{3}{2}}}{16}$

$\doteq 0.1768$

Question 7, page 181

a. The error was made in subtracting the exponents. The power should read: $t^{1.2-(-0.5)}$ and the correct answer is $t^{1.7}$.

b. The error was made in evaluating $16^{0.5}$. The value of $16^{0.5} = 16^{\frac{1}{2}} = \sqrt{16} = \sqrt{4^2} = 4$, not 8. The correct answer is 4*x*.

Question 11, page 182

a. 5 years and 3 months = 5.25 years. Substitute $n = 5.25$ into the equation.

$V = 10\,000(0.88)^n$

$V = 10\,000(0.88)^{5.25}$

$V \doteq 5\,111.33$

The value of the mutual fund will be approximately $5 111.33.

b. For 3.5 years ago, substitute $n = -3.5$ into the equation.

$$V = 10\,000(0.88)^n$$
$$V = 10\,000(0.88)^{-3.5}$$
$$V \doteq 15\,642.66$$

The value of the mutual fund 3.5 years ago was approximately $15 642.66.

Question 1, page 192

a. $4^{\frac{3}{2}} = (\sqrt{4})^3$

b. $32^{\frac{1}{5}} = \sqrt[5]{32}$

c. $64^{0.5} = 64^{\frac{1}{2}}$
$= \sqrt{64}$

d. $\left(\frac{1}{100}\right)^{\frac{1}{4}} = \sqrt[4]{\frac{1}{100}}$

e. $\left(\frac{y^4}{x^3}\right)^{\frac{1}{3}} = \sqrt[3]{\frac{y^4}{x^3}}$

f. $(m^n)^{\frac{3}{2}} = (\sqrt{m^n})^3$

Question 2, page 192

a. $\sqrt{(12p)^3} = (12p)^{3\cdot\frac{1}{2}}$
$= (12p)^{\frac{3}{2}}$

b. $\sqrt[5]{5^3} = (5^3)^{\frac{1}{5}}$
$= 5^{3\cdot\frac{1}{5}}$
$= 5^{\frac{3}{5}}$

c. $\sqrt[4]{x^3} = (x^3)^{\frac{1}{4}}$
$= x^{3\cdot\frac{1}{4}}$
$= x^{\frac{3}{4}}$

d. $\sqrt[3]{\frac{s^3}{t^5}} = \left(\frac{s^3}{t^5}\right)^{\frac{1}{3}}$
$= \frac{s^{3\cdot\frac{1}{3}}}{t^{5\cdot\frac{1}{3}}}$
$= \frac{s}{t^{\frac{5}{3}}}$

e. $\sqrt{y^{\frac{5}{3}}} = \left(y^{\frac{5}{3}}\right)^{\frac{1}{2}}$
$= y^{\frac{5}{3}\cdot\frac{1}{2}}$
$= y^{\frac{5}{6}}$

f. $\sqrt[n]{8} = (8)^{\frac{1}{n}}$

Question 3, page 192

a. $\sqrt{0.36} = 0.6$

b. $27^{\frac{1}{3}} = \sqrt[3]{27}$
$= \sqrt[3]{3 \times 3 \times 3}$
$= \sqrt[3]{3^3}$
$= 3$

c. $4\sqrt{17} \doteq 16.4924$

d. $(65)^{\frac{2}{3}} = (\sqrt[3]{65})^2$
$\doteq 16.1662$

e. $0.3(22)^{\frac{1}{2}} = 0.3(\sqrt{22})$
$\doteq 1.4071$

f. $\frac{\sqrt{36}}{\sqrt{7}} = \frac{6}{\sqrt{7}}$

$\doteq 2.2678$

Question 11, page 193

$l = 0.46\sqrt[3]{m}$

$l = 0.46\sqrt[3]{25}$

$l = 1.3450...$

The length of a 25 kg Pacific halibut is approximately 1.35 m.

Question 12, page 193

$v = \sqrt{30df}$

$v = \sqrt{30 \cdot 75 \cdot 0.7}$

$v = 39.6862.....$

The speed of the vehicle was approximately 40 mph.

Question 15, page 194

$l = \frac{8t^2}{\pi^2}$

$l = \frac{8(2)^2}{\pi^2}$

$l = 3.2422...$

The pendulum is approximately 3.24 ft. long.

Question 16, page 194

$\frac{500}{580} = \frac{\cancel{580}}{\cancel{580}}\sqrt[3]{8p}$

$\frac{25}{29} = \sqrt[3]{8p}$

$\left(\frac{25}{29}\right)^3 = \left(\sqrt[3]{8p}\right)^3$

$\frac{25^3}{29^3} = 8p$

$\frac{15\,625}{24\,389} = 8p$

$\frac{\frac{15\,625}{24\,389}}{8} = \frac{\cancel{8}p}{\cancel{8}}$

$0.0800... = p$

The store should offer a sales discount of approximately 8%.

Question 19, page 195

a. $\sqrt{\sqrt{2^{\frac{4}{5}}}} = \sqrt{2^{\left(\frac{4}{5}\cdot\frac{1}{2}\right)}}$
$= \sqrt{2^{\left(\frac{4}{10}\right)}}$
$= 2^{\left(\frac{4}{10}\cdot\frac{1}{2}\right)}$
$= 2^{\frac{4}{20}}$
$= 2^{\frac{1}{5}}$

b. $\sqrt[4]{\sqrt{256}} = 256^{\left(\frac{1}{2}\cdot\frac{1}{4}\right)}$
$= 256^{\frac{1}{8}}$

Question 20, page 195

$\sqrt[4]{(-x)^4} = x$

The result of taking the fourth root of a number raised to the power of 4 will always be positive. As such, the statement is only true when $x \geq 0$.