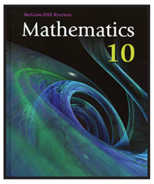
Enhance Your Understanding

Lesson 5.1: Polynomial Multiplication

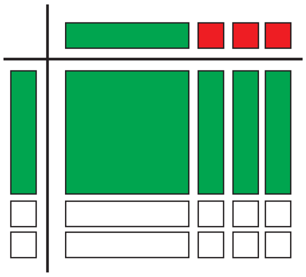


Refer to page 209 in *Mathematics 10* for more practice.

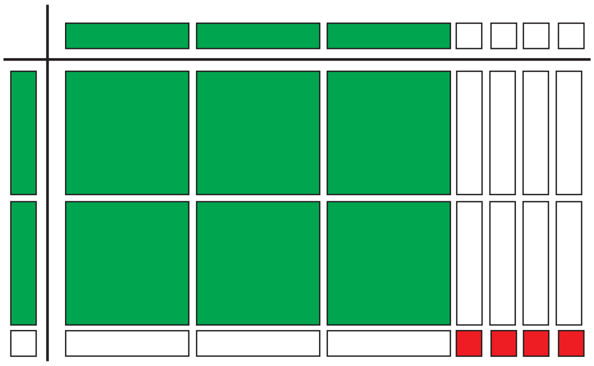
Page 209, #1a, 1b, 2, 3a, 3b, 3f, 4a, 4f, 6a, 6b, 11, 12, and 14

Question 1, page 209

a. $(x-2)(x+3)=x^2+x-6$

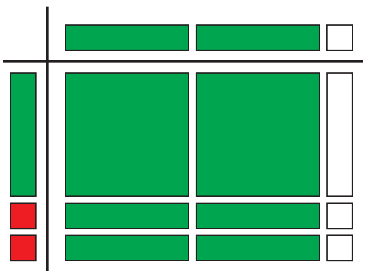


b. $(3x-4)(2x-1)=6x^2-11x+4$



Question 2, page 209

a. There are two positive x^2-tiles, four positive x-tiles, one negative x-tile, and two negative 1-tiles. Therefore, the product is $2x^2+3x-2$.



b. The dimensions of the model are $(2x-1)$ by $(x+2)$.

Question 3, page 209

a. $$\begin{aligned}(x+5)(x-2)&=(x)(x)+(x)(-2)+(5)(x)+(5)(-2)\\&=x^2-2x+5x-10\\&=x^2+3x-10\end{aligned}$$

b. $$\begin{aligned}(x-3)^2&=(x-3)(x-3)\\&=(x)(x)+(x)(-3)+(-3)(x)+(-3)(-3)\\&=x^2-3x-3x+9\\&=x^2-6x+9\end{aligned}$$

f. $$\begin{aligned}(4j+2k)(6j-3k)&=(4j)(6j)+(4j)(-3k)+(2k)(6j)+(2k)(-3k)\\&=24j^2-12jk+12jk-6k^2\\&=24j^2-6k^2\end{aligned}$$

Question 4, page 209

a. $$\begin{aligned}x(3x^2-5x+8)&=(x)(3x^2)+(x)(-5x)+(x)(8)\\&=3x^3-5x^2+8x\end{aligned}$$

f. $$\begin{aligned}(2y^2+3y-1)(y^2+4y+5)&=(2y^2)(y^2)+(2y^2)(4y)+(2y^2)(5)+(3y)(y^2)+(3y)(4y)\\&\quad+(3y)(5)+(-1)(y^2)+(-1)(4y)+(-1)(5)\\&=2y^4+8y^3+10y^2+3y^3+12y^2+15y-y^2-4y-5\\&=2y^4+11y^3+21y^2+11y-5\end{aligned}$$

Question 6, page 210

a. $$\begin{aligned}(4n+2)+(2n-3)(3n-2)&=(4n+2)+(2n)(3n)+(2n)(-2)+(-3)(3n)+(-3)(-2)\\&=(4n+2)+6n^2-4n-9n+6\\&=4n+2+6n^2-4n-9n+6\\&=6n^2-9n+8\end{aligned}$$

b. $(f+7)(2f-4)-(3f+1)^2 = (f)(2f)+(f)(-4)+(7)(2f)+(7)(-4)-(3f+1)(3f+1)$
$= 2f^2-4f+14f-28-[(3f)(3f)+(3f)(1)+(1)(3f)+(1)(1)]$
$= 2f^2+10f-28-[9f^2+3f+3f+1]$
$= 2f^2+10f-28-9f^2-3f-3f-1$
$= -7f^2+4f-29$

Question 11, page 211

diameter = $6x + 4$, radius = $\dfrac{6x+4}{2} = \dfrac{2(3x+2)}{2} = 3x+2$

To find the area of a circle, use the formula $A = \pi r^2$.

$A = \pi r^2$
$A = \pi(3x+2)^2$
$= \pi(3x+2)(3x+2)$
$= \pi[(3x)(3x)+(3x)(2)+(2)(3x)+(2)(2)]$
$= \pi[9x^2+6x+6x+4]$
$= \pi[9x^2+12x+4]$
$= (9x^2)(\pi)+(12x)(\pi)+(4)(\pi)$
$= 9\pi x^2+12\pi x+4\pi$

Question 12, page 211

a. Bryan made an error in step 3. He added p^2 and $10p$ to get $11p^2$.

b. Using the equation $(2p-3)(p+4) = 2p^2-5p-12$, substitute $p = 1$.

Left Side	Right Side
$(2(1)-3)(1+4)$	$2(1)^2-5(1)-12$
$(2-3)(5)$	$2-5-12$
$(-1)(5)$	-15
-5	
Left Side ≠ Right Side	

Since the left side is not equal to the right side, the given equation is not true.

Question 14, page 212

a. Let x represent the side length of the square rug, in feet. The dimensions of the new rectangular rug are $(x+2)$ by $(x-1)$.

b. To find the area of the new rectangular rug, use the formula $A = lw$.

$$\begin{aligned} A &= lw \\ &= (x+2)(x-1) \\ &= x^2 - x + 2x - 2 \\ &= x^2 + x - 2 \end{aligned}$$

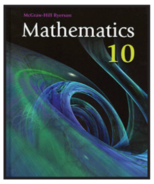
c. The area of the square rug is $(3\text{ ft})(3\text{ ft}) = 9\text{ ft}^2$.

To find the area of the new rug, substitute $x = 3$ into the expression for the area of the new rug.

$$\begin{aligned} A_{\text{rectangle}} &= (x+2)(x-1) \\ &= (3+2)(3-1) \\ &= (5)(2) \\ &= 10 \end{aligned}$$

The area of the new rug is 10 ft^2. The rectangular rug's area is 1 ft^2 greater than the square rug's area.

Lesson 5.2: Common Factors of Polynomials



Refer to page 220 in *Mathematics 10* for more practice.

Page 220, #1a, 1b, 2c, 2d, 4a, 4e, 5, 6a, 6c, 6e, 7a, 7c, 7e, 11a, 12b, 12c, 12d, and 16

Question 1, page 220

a. Factors of 20: $\boxed{1}, \boxed{2}, 4, \boxed{5}, \boxed{10}, 20$

Factors of 30: $\boxed{1}, \boxed{2}, 3, \boxed{5}, 6, \boxed{10}, 15, 30$

The GCF of 20 and 30 is 10.

b. Factors of 28: $\boxed{1}, \boxed{2}, \boxed{4}, 7, 14, 28$

Factors of 40: $\boxed{1}, \boxed{2}, \boxed{4}, 5, 8, 10, 20, 40$

The GCF of 28 and 40 is 4.

Question 2, page 220

c. Factors of 81: $\boxed{1}, \boxed{3}, \boxed{9}, \boxed{27}, 81$

Factors of 54: $\boxed{1}, 2, \boxed{3}, 6, \boxed{9}, 18, \boxed{27}, 54$

The GCF of 81 and 54 is 27.

d. Factors of 256: $\boxed{1}, \boxed{2}, 4, 8, 16, 32, 64, 128, 256$

Factors of 216: $\boxed{1}, \boxed{2}, 3, 4, 6, 8, 9, 12, 18, 24, 27, 36, 54, 72, 108, 216$

Factors of 78: $\boxed{1}, \boxed{2}, 3, 6, 13, 26, 39, 78$

The GCF of 256, 216, and 78 is 2.

Question 4, page 220

a. Factors of 15: $\boxed{1}, \boxed{3}, 5, 15$

Factors of 18: $\boxed{1}, 2, \boxed{3}, 6, 9, 18$

The numerical GCF is 3.

Factors of a^2b: $\boxed{a}, a^2, \boxed{b}$

Factors of ab: $\boxed{a}, \boxed{b}$

The variable GCF is ab.

The GCF of $15a^2b$ and $18ab$ is $3ab$.

e. Factors of 14: $\boxed{1}, 2, 7, 14$

Factors of 24: $\boxed{1}, 2, 3, 4, 6, 8, 12, 24$

Factors of 7: $\boxed{1}, 7$

The numerical GCF is 1.

Factors of p^4q^5: $p, p^2, \boxed{p^3}, p^4, q, q^2, \boxed{q^3}, q^4, q^5$

Factors of p^5q^4: $p, p^2, \boxed{p^3}, p^4, p^5, q, q^2, \boxed{q^3}, q^4$

Factors of p^3q^3: $p, p^2, \boxed{p^3}, q, q^2, \boxed{q^3}$

The variable GCF is p^3q^3.

The GCF of $14p^4q^5, -24p^5q^4$, and $7p^3q^3$ is p^3q^3.

Question 5, page 221

a. The GCF is 5.

$$\begin{aligned} 5x + 15 &= (5)(x) + (5)(3) \\ &= 5(x+3) \end{aligned}$$

b. The GCF is y.

$$\begin{aligned} 3y^2 - 5y &= (3)(y)(y) - (5)(y) \\ &= y(3y-5) \end{aligned}$$

c.
$$\begin{aligned} w^2x &= w \bullet w \bullet x \\ w^2y &= w \bullet w \bullet y \\ -w^2z &= -1 \bullet w \bullet w \bullet z \end{aligned}$$

The GCF is $w \bullet w = w^2$.

$$\frac{w^2x}{w^2} = x \qquad \frac{w^2y}{w^2} = y \qquad \frac{-w^2z}{w^2} = -z$$

$$w^2x = (w^2)(x) \qquad w^2y = (w^2)(y) \qquad -w^2z = (w^2)(-z)$$

$$w^2x + w^2y - w^2z = w^2(x + y - z)$$

d. $6a^3b = 2 \cdot 3 \cdot a \cdot a \cdot a \cdot b$

$-18ab^2 = -1 \cdot 2 \cdot 3 \cdot 3 \cdot a \cdot b \cdot b$

The GCF is $2 \cdot 3 \cdot a \cdot b = 6ab$.

$\frac{6a^3b}{6ab} = a^2$ $\frac{-18ab^2}{6ab} = -3b$

$6a^3b = (6ab)(a^2)$ $-18ab^2 = (6ab)(-3b)$

$6a^3b - 18ab^2 = 6ab(a^2 - 3b)$

e. $9x^3 = 3 \cdot 3 \cdot x \cdot x \cdot x$

$-12x^2 = -1 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x$

$6x = 2 \cdot 3 \cdot x$

The GCF is $3 \cdot x = 3x$.

$\frac{9x^3}{3x} = 3x^2$ $\frac{-12x^2}{3x} = -4x$ $\frac{6x}{3x} = 2$

$9x^3 = (3x)(3x^2)$ $-12x^2 = (3x)(-4x)$ $6x = (3x)(2)$

$9x^3 - 12x^2 + 6x = 3x(3x^2 - 4x + 2)$

Question 6, page 221

a. $\frac{6a^2bc}{2ac} = 3ab$ $\frac{9ab^2}{3b} = 3ab$

The unknown factor is $3ab$.

c. $\frac{3d^2}{3d} = d$ $\frac{-21d}{3d} = -7$

The unknown factor is $(d - 7)$.

e. $\frac{12x^2y^2}{3xy} = 4xy$ $\frac{-16xy}{-4} = 4xy$

The unknown factor is $4xy$.

Question 7, page 221

a. The GCF is $(y-2)$.

$$\frac{3y(y-2)}{(y-2)} = 3y \qquad \frac{4(y-2)}{(y-2)} = 4$$

$$3y(y-2) + 4(y-2) = (y-2)(3y+4)$$

c. $$\begin{aligned} 2cx - 8x + 7c - 28 &= (2cx - 8x) + (7c - 28) \\ &= 2x(c-4) + 7(c-4) \\ &= (c-4)(2x+7) \end{aligned}$$

e. $$\begin{aligned} 2y^4 + y^3 - 10y - 5 &= (2y^4 + y^3) + (-10y - 5) \\ &= (y^3)(2y+1) + (-5)(2y+1) \\ &= (2y+1)(y^3 - 5) \end{aligned}$$

Question 11, page 222

a. Answer may vary. For example, $6x^2 + 18x$.

$$\frac{6x^2}{6x} = x \qquad \frac{18x}{6x} = 3$$

The monomial $6x$ divides evenly into each term.

Question 12, page 222

b. The second factor, $(5x)$, is incorrect. The answer should be $5x(x-2) - (x-2) = (x-2)(5x-1)$.

c. The GCF of $9a^2b^2$ was not used. The answer should be $9a^2b^3 - 27a^2b^2 + 81a^3b^3 = 9a^2b^2(b - 3 + 9ab)$.

d. The greatest common factor was not factored out of each term and the grouping process was left incomplete. The answer should be as follows.

$$\begin{aligned} 4fx + 16f + 2x + 8 &= 2[2fx + 8f + x + 4] \\ &= 2[(2fx + 8f) + (x+4)] \\ &= 2[2f(x+4) + 1(x+4)] \\ &= 2(x+4)(2f+1) \end{aligned}$$

Question 16, page 223

Answer may vary.

For example, find a common factor of $15x^2$ and $30x$.

Factors of $15x^2$: $1, 3, 5, 15, x, x^2$

Factors of $30x$: $1, 2, 3, 5, 6, 10, 15, 30, x$

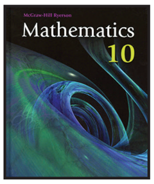
The GCF of $15x^2$ and $30x$ is $15x$.

$\frac{15x^2}{15x} = x$ $\qquad$ $\frac{30x}{15x} = 2$

The length could be $(x + 2)$ and the width could be $15x$.

Other pairs of factors, neither of which being the GCF, would also be acceptable.

Lesson 5.3: Factoring Trinomials



Refer to page 234 in *Mathematics 10* for more practice.

Page 234, #1, 2, 3a, 3b, 4a, 4c, 4e, 6a, 6c, 6e, 9a, 9b, and 13

Question 1, page 234

a. There are one positive x^2-tile, four positive x-tiles, and three positive 1-tiles. The trinomial is $x^2 + 4x + 3$. The dimensions of the rectangle are $(x + 1)$ and $(x + 3)$.

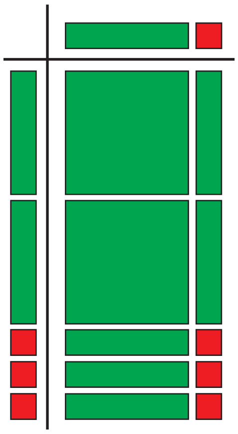
b. There are one positive x^2-tile, two positive x-tiles, and one positive 1-tile. The trinomial is $x^2 + 2x + 1$. The dimensions of the rectangle are $(x + 1)$ and $(x + 1)$, or $(x + 1)^2$.

c. There are one positive x^2-tile, two positive x-tiles, one negative x-tile, and two negative 1-tiles. The trinomial is $x^2 + x - 2$. The dimensions of the rectangle are $(x + 2)$ and $(x - 1)$.

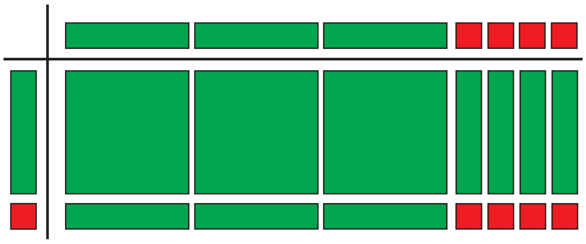
d. There are one positive x^2-tile, five positive x-tiles, and four positive 1-tiles. The trinomial is $x^2 + 5x + 4$. The dimensions of the rectangle are $(x + 4)$ and $(x + 1)$.

Question 2, page 234

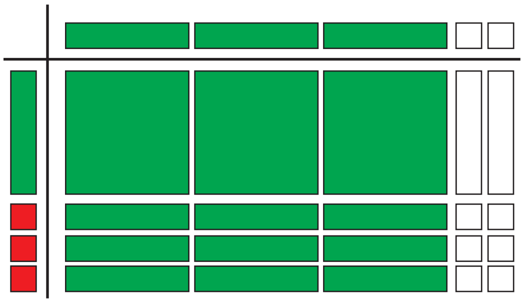
a.



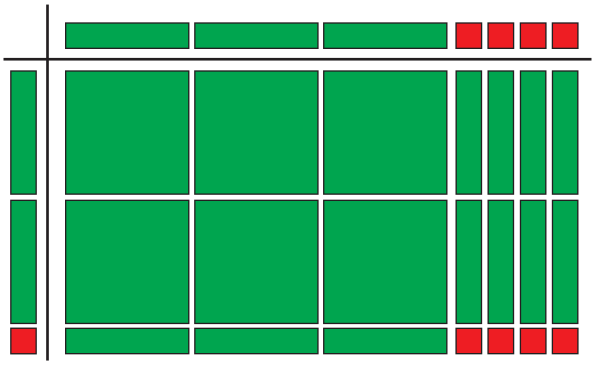
b.



c.



d.



Question 3, page 234

a.

First Integer	Second Integer	Product	Sum
1	45	$1 \times 45 = 45$	$1 + 45 = 46$
3	15	$3 \times 15 = 45$	$3 + 15 = 18$
5	9	$5 \times 9 = 45$	$5 + 9 = 14$

The integers 5 and 9 have a product of 45 and a sum of 14.

b.

First Integer	Second Integer	Product	Sum
–1	–6	$-1 \times (-6) = 6$	$-1 + -6 = -7$
–2	–3	$-2 \times (-3) = 6$	$-2 + -3 = -5$

The integers –2 and –3 have a product of 6 and a sum of –5.

Question 4, page 234

a. $b = 7$ and $c = 10$

The integers 2 and 5 add to 7 and have a product of 10.

$x^2 + 7x + 10 = (x + 2)(x + 5)$

c $b = 5$ and $c = 4$

The integers 1 and 4 add to 5 and have a product of 4.

$k^2 + 5k + 4 = (k + 1)(k + 4)$

e. $b = 10$ and $c = 24$

The integers 4 and 6 add to 10 and have a product of 24.

$d^2 + 10d + 24 = (d + 4)(d + 6)$

Question 6, page 235

a. $ac = 10$ and $b = 7$

The integers 2 and 5 have a product of 10 and a sum of 7.

$$\begin{aligned} 2x^2 + 7x + 5 &= 2x^2 + (2 + 5)x + 5 \\ &= 2x^2 + 2x + 5x + 5 \\ &= (2x^2 + 2x) + (5x + 5) \\ &= 2x(x + 1) + 5(x + 1) \\ &= (x + 1)(2x + 5) \end{aligned}$$

c. $ac = 24$ and $b = 10$

The integers 4 and 6 have a product of 24 and a sum of 10.

$$\begin{aligned} 3m^2 + 10m + 8 &= 3m^2 + (4 + 6)m + 8 \\ &= 3m^2 + 4m + 6m + 8 \\ &= (3m^2 + 4m) + (6m + 8) \\ &= m(3m + 4) + 2(3m + 4) \\ &= (3m + 4)(m + 2) \end{aligned}$$

e. $ac = 72$ and $b = 17$

The integers 8 and 9 have a product of 72 and a sum of 17.

$$\begin{aligned} 12q^2 + 17q + 6 &= 12q^2 + (8 + 9)q + 6 \\ &= 12q^2 + 8q + 9q + 6 \\ &= (12q^2 + 8q) + (9q + 6) \\ &= 4q(3q + 2) + 3(3q + 2) \\ &= (3q + 2)(4q + 3) \end{aligned}$$

Question 9, page 235

a. Find two pairs of positive factors of 12. The sum of each pair gives the value of b.

The integers 3 and 4 are factors of 12. $3 + 4 = 7$

The integers 2 and 6 are factors of 12. $2 + 6 = 8$

Two values of b that allow the expression to be factored are 7 and 8.

b. Find two pairs of positive factors of 4. The sum of each pair gives the value of *b*.

The integers 1 and 4 are factors of 4. $1 + 4 = 5$

The integers 2 and 2 are factors of 4. $2 + 2 = 4$

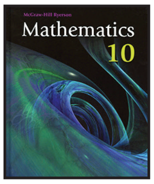
Two values of *b* that allow the expression to be factored are 4 and 5.

Question 13, page 236

a. Answer may vary. The trinomial $5x^2 + x + 16$ cannot be factored.

b. It cannot be factored because no two integers multiply to give 80 and add to give 1.

Lesson 5.4: Other Factoring Strategies



Refer to page 246 in *Mathematics 10* for more practice.

Page 246, #1, 4b, 4c, 5a, 5c, 5e, 6a, 6c, 6e, 8a, and 14

Question 1, page 246

a. The factors are $(x - 2)(x + 2)$.

b. The factors are $(2x - 3)(2x + 3)$.

c. The factors are $(x + 4)(x + 4)$.

d. The factors are $(x - 3)(x - 3)$.

Question 4, page 247

b. $\sqrt{16r^6} = \sqrt{4^2(r^3)^2} = 4r^3$ and $\sqrt{81} = \sqrt{9^2} = 9$

$16r^6 - 81 = (4r^3 - 9)(4r^3 + 9)$

c. $x^2 - 12x + 36 = x^2 + 2(x)(-6) + (-6)^2$

$= (x - 6)^2$

Question 5, page 247

a $\sqrt{x^2} = x$ and $\sqrt{16} = \sqrt{4^2} = 4$

$x^2 - 16 = (x-4)(x+4)$

c. The binomial $w^2 + 169$ cannot be factored. It represents a sum of squares.

e. $\sqrt{36c^2} = \sqrt{6^2c^2} = 6c$ and $\sqrt{49d^2} = \sqrt{7^2d^2} = 7d$

$36c^2 - 49d^2 = (6c - 7d)(6c + 7d)$

Question 6, page 247

a. $x^2 + 12x + 36 = x^2 + 2(x)(6) + 6^2$
$= (x+6)^2$

c. The trinomial is not a perfect square trinomial, nor is it in the form of $(ax)^2 - 2abx + b^2$. There are no two integers that give a product of –144 and a sum of –24. Therefore, the trinomial cannot be factored over the integers.

e. $16k^2 - 8k + 1 = 16k^2 + 2(4k)(-1) + (-1)^2$
$= (4k-1)^2$

Question 8, page 247

a. The middle term is twice the product of the square root of the first term and the square root of the last term.

$n = (2)\sqrt{(1)(25)}$
$n = (2)(\pm 5)$
$n = \pm 10$

The two possible values of n are 10 and –10.

The factored trinomials are $x^2 + 10x + 25 = (x+5)^2$ and $x^2 - 10x + 25 = (x-5)^2$.

Question 14, page 248

a. Shaded Area = Area of Outer Circle – Area of Inner Circle

$$\begin{aligned}\text{Shaded Area} &= \pi(r+4)^2 - \pi r^2\\ &= \pi(r+4)(r+4) - \pi r^2\\ &= \pi[r^2+4r+4r+16] - \pi r^2\\ &= \pi[r^2+8r+16] - \pi r^2\\ &= \pi r^2 + 8\pi r + 16\pi - \pi r^2\\ &= 8\pi r + 16\pi\end{aligned}$$

b. The GCF is 8π.

$$8\pi r + 16\pi = 8\pi(r+2)$$

c. Substitute $r = 6$ into $8\pi(r+2)$

$$\begin{aligned}\text{Shaded Area} &= 8\pi(r+2)\\ &= 8\pi(6+2)\\ &= 8\pi(8)\\ &= 201.061\ldots\end{aligned}$$

The area of the shaded region is approximately $201.1\ \text{cm}^2$.