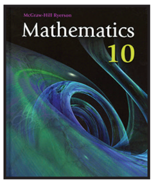
Enhance Your Understanding

Lesson 8.1: Systems of Linear Equations and Graphs



Page 426, #1, 3a , 5a, 5c, 7, 10, 12, and 15

Page 454, #1, 2, 3, 4, 7, 10, 11, and 12

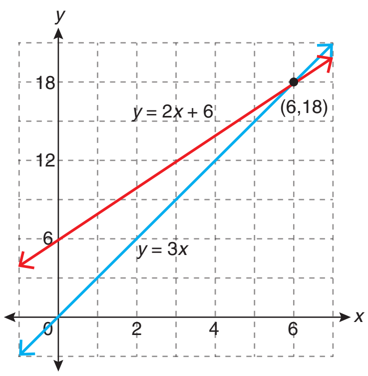
Question 1, page 426

Yes, the table of values shows that when $x = 1$, the value of each relation is 3. The graph shows the intersection of the two lines at point (1,3). Both systems have the same solution.

Question 3, page 427

a.

x	$y = 2x + 6$	$y = 3x$
0	$6 = 2(0) + 6$	$0 = 3(0)$
1	$8 = 2(1) + 6$	$3 = 3(1)$
2	$10 = 2(2) + 6$	$6 = 3(2)$
3	$12 = 2(3) + 6$	$9 = 3(3)$
4	$14 = 2(4) + 6$	$12 = 3(4)$
5	$16 = 2(5) + 6$	$15 = 3(5)$
6	$18 = 2(6) + 6$	$18 = 3(6)$
7	$20 = 2(7) + 6$	$21 = 3(7)$



Question 5, page 426

a.

Left Side	Right Side
y 7	$3x-5$ $3(4)-5$ 7
LS = RS	

Left Side	Right Side
y 7	$11-x$ $11-4$ 7
LS = RS	

The point (4,7) is the solution to the system of equations.

c.

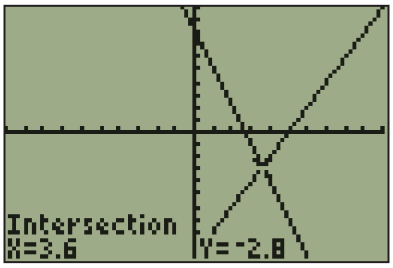
Left Side	Right Side
$2x-3y$ $2(-6)-3(-10)$ 18	18
LS = RS	

Left Side	Right Side
$x+2y$ $(-6)+2(-10)$ -26	-26
LS = RS	

The point (–6,–10) is the solution to the system of equations.

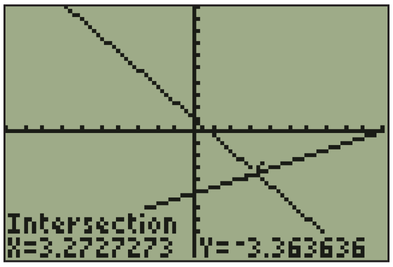
Question 7, page 427

a.



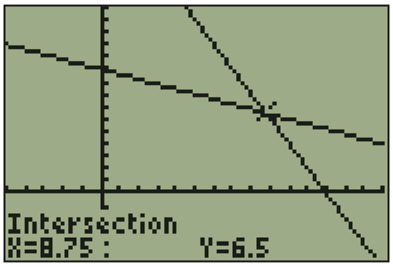
The point of intersection occurs at (3.6,–2.8), so this is the solution to the system of equations.

b.



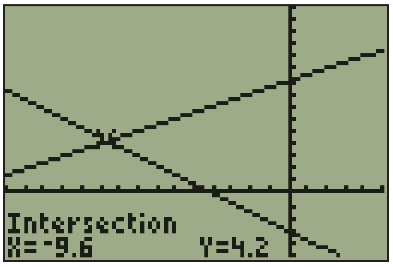
The point of intersection occurs at (3.2727…, –3.3636…), so this is the solution to the system of equations.

c.



The point of intersection occurs at (8.75,6.5), so this is the solution to the system of equations.

d.



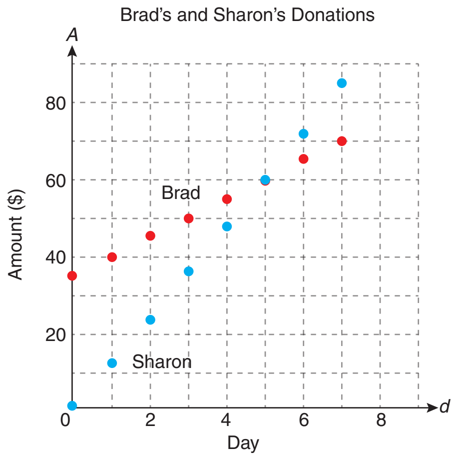
The point of intersection occurs at (–9.6,4.2), so this is the solution to the system of equations.

Question 10, page 428

a.

Brad	
Day	Amount ($)
0	35
1	40
2	45
3	50
4	55
5	60
6	65
7	70

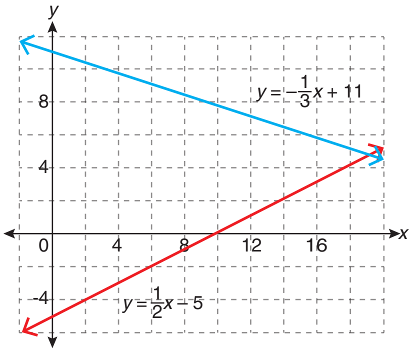
Sharon	
Day	Amount ($)
0	0
1	12
2	24
3	36
4	48
5	60
6	72
7	84



b. The solution to the system is (5,60), which represents the number of days it will take Brad and Sharon to collect the same amount of money.

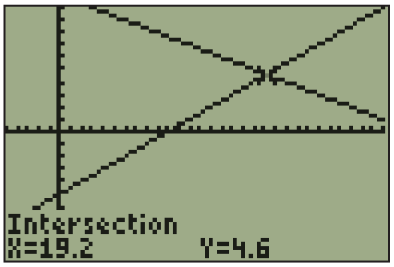
Question 12, page 428

a.



The point of intersection occurs at approximately (19,4.5).

b.



The point of intersection occurs at (19.2,4.6), so this is the solution to the system of equations.

c. The graphing-by-hand method works well when the lines meet where lines of the grid meet. However, in this case, the solution must be estimated. Graphing using technology allows for a quick, accurate solution. Sometimes a window adjustment is needed to display the appropriate area for the point of intersection, which can add a challenge.

Question 15, page 429

a. Brendan began with a 400 m head start but ran more slowly than Malcolm did. Brendan took 9 min to complete 2 800 m. Malcolm made up the difference by running 3 200 m in 8 min. He passed Brendan 4.5 min into the run, after he had run about 1 800 m.

b. Since each person is running at a constant speed during this portion of the run, a system of linear equations can be used.

Question 1, page 454

a. A and B, A and C, C and D, A and D, and B and D are pairs of lines that form linear systems with one solution.

b. B and C are parallel, so they form a linear system with no solution.

Question 2, page 455

a. The equations have the same slope and the same y-intercept, so the lines are coincident and the linear system will have an infinite number of solutions.

b. The two lines have different slopes, so they will have one point of intersection and the linear system will have one solution.

c. The two lines have the same slope but different y-intercepts, so they are parallel lines. Parallel lines have no points of intersection, so this linear system has no solution.

Question 3, page 455

a. Rearrange the first equation into slope-intercept form.

$$x + 3y = 6$$
$$3y = 6 - x$$
$$y = -\frac{1}{3}x + 2$$

$$y = -\frac{1}{3}x + 6$$

The two lines have the same slope, but different y-intercepts, so they are parallel lines. Parallel lines have no points of intersection, so this linear system has no solution.

b. Rearrange each equation into slope-intercept form.

$$\begin{aligned} 3x - y &= 12 \\ -y &= 12 - 3x \\ y &= -12 + 3x \\ y &= 3x - 12 \end{aligned} \qquad \begin{aligned} 4x - y &= 12 \\ -y &= 12 - 4x \\ y &= -12 + 4x \\ y &= 4x - 12 \end{aligned}$$

The two lines have different slopes, so they will have one point of intersection and the linear system will have one solution.

c. Rearrange each equation into slope-intercept form.

$$\begin{aligned} x - 4y &= 8 \\ -4y &= -x + 8 \\ y &= \frac{1}{4}x - 2 \end{aligned} \qquad \begin{aligned} x + 4y &= 20 \\ 4y &= -x + 20 \\ y &= -\frac{1}{4}x + 5 \end{aligned}$$

The two lines have different slopes, so they will have one point of intersection and the linear system will have one solution.

Question 4, page 455

a. The graph of a system of linear equations with no solution would be one of parallel lines. The equations of these lines would have the same slope, but different y-intercepts.

b. The graph of a system of linear equations with one solution would be one of lines that meet at one point. The equations of these lines would have different slopes.

c. For a system of linear equations to have an infinite number of solutions, the two equations can be manipulated into exactly the same equation. The graph of this type of linear system appears as a single line (one line on top of the other).

Question 7, page 455

Let E represent earnings, in dollars, and let s represent the number of subscriptions sold.

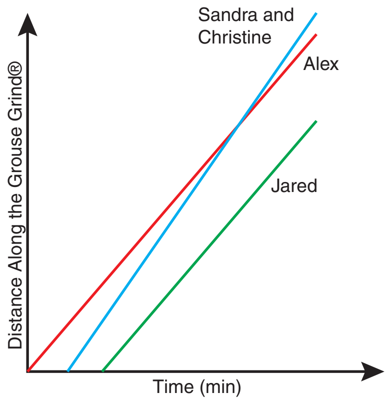
a. The equation will be $E = 360 + 8.25s$ for both Brian and Charlie. This means that there are an infinite number of solutions to the linear system, and the two will always have the same earnings.

b. The equations will be $E = 472 + 7s$ for Alyssa and $E = 360 + 8.25s$ for Brian. Since the two equations have different slopes, there will be one solution to the linear system. Because the slope of Brian's equation is higher, he will eventually catch and pass Alyssa in earnings.

c. The equations will be $E = 360 + 8.25s$ for Charlie and $E = 413 + 8.25s$ for Dena. Since the two equations have the same slope, there will be no solution to the linear system. This means that Dena will always earn $53 more than Charlie.

Question 10, page 456

a.



b. The linear system for Alex and Jared has no solution. They will always remain the same distance apart on the climb.

The linear system for Sandra and Christine has an infinite number of solutions. They will always be walking together for the length of the Grind.

The linear systems for Sandra and Alex, and for Christine and Alex will each have one solution. Sandra and Christine will catch and pass Alex (assuming they walk fast enough to pass him before Alex reaches the top).

The linear systems for Sandra and Jared, and for Christine and Jared will have no solution within the given context. Sandra and Christine will always be ahead of Jared and they will steadily increase their lead.

Question 11, page 457

The statement is false. There are an infinite number of solutions, but they are a distinct set of collinear points. There are many points in the coordinate plane that are not on any of the lines in the particular system.

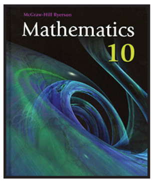
Question 12, page 457

a. No. When the slopes are the same, there are two possible outcomes: no solution if the lines are parallel, and an infinite number of solutions if the lines are coincident.

b. No. A prediction is not possible because the two lines could have one point of intersection with the *y*-intercept being the solution, or the two lines could be coincident with the *y*-intercept being just one of an infinite number of solutions.

c. Yes. Since the lines are coincident, the system will have an infinite number of solutions.

Lesson 8.2: Solving Systems of Linear Equations by Substitution



Page 474, #1, 2, 4, 6, 7, and 8

Question 1, page 474

a. $y = 3x + 2$ and $x + y = 14$

$$\begin{aligned} x + y &= 14 \\ x + 3x + 2 &= 14 \\ 4x + 2 &= 14 \\ 4x &= 14 - 2 \\ 4x &= 12 \\ \frac{\cancel{4}x}{\cancel{4}} &= \frac{12}{4} \\ x &= 3 \end{aligned}$$

$$\begin{aligned} y &= 3x + 2 \\ y &= 3(3) + 2 \\ y &= 11 \end{aligned}$$

The solution is (3,11).

b. $y = -3x$ and $y - x = 24$

$$
\begin{aligned}
y - x &= 24 \\
-3x - x &= 24 \\
-4x &= 24 \\
\frac{\cancel{-4}x}{\cancel{-4}} &= \frac{24}{-4} \\
x &= -6
\end{aligned}
$$

$$
\begin{aligned}
y &= -3x \\
y &= -3(-6) \\
y &= 18
\end{aligned}
$$

The solution is (–6,18).

c. $y = x - 7$ and $x + y = 17$

$$
\begin{aligned}
x + y &= 17 \\
x + x - 7 &= 17 \\
2x - 7 &= 17 \\
2x &= 24 \\
\frac{\cancel{2}x}{\cancel{2}} &= \frac{24}{2} \\
x &= 12
\end{aligned}
$$

$$
\begin{aligned}
y &= x - 7 \\
y &= 12 - 7 \\
y &= 5
\end{aligned}
$$

The solution is (12,5).

Question 2, page 474

a. $2x - 3y = 10$ and $x + y = 0$

$$x + y = 0$$
$$x = -y$$

$$2x - 3y = 10$$
$$2(-y) - 3y = 10$$
$$-2y - 3y = 10$$
$$-5y = 10$$
$$\frac{\cancel{-5}y}{\cancel{-5}} = \frac{10}{-5}$$
$$y = -2$$

$$x = -y$$
$$x = -(-2)$$
$$x = 2$$

The solution is (2,–2).

b. $m = 8j$ and $-m + 2 = -7j$

$$-m + 2 = -7j$$
$$-(8j) + 2 = -7j$$
$$2 = -7j + 8j$$
$$2 = j$$

$$m = 8j$$
$$m = 8(2)$$
$$m = 16$$

The solution is (16,2).

c. $2k = 6n + 9$ and $n - 2k = -4$

$$n - 2k = -4$$
$$n = -4 + 2k$$

$$2k = 6n + 9$$
$$2k = 6(-4 + 2k) + 9$$
$$2k = -24 + 12k + 9$$
$$2k = -15 + 12k$$
$$\frac{\cancel{-10}k}{\cancel{-10}} = \frac{-15}{-10}$$
$$k = 1.5$$

$$n = -4 + 2k$$
$$n = -4 + 2(1.5)$$
$$n = -1$$

The solution is (–1,1.5).

Question 4, page 475

a. $y = \frac{1}{3}x - 5$ and $x - \frac{y}{5} = 13$

$$5(x) - 5\left(\frac{y}{5}\right) = 5(13)$$
$$5x - y = 65$$
$$5x - \left(\frac{1}{3}x - 5\right) = 65$$
$$5x - \frac{1}{3}x + 5 = 65$$
$$\frac{14}{3}x + 5 = 65$$
$$\frac{14}{3}x = 60$$
$$\frac{\cancel{3}}{\cancel{14}}\left(\frac{\cancel{14}}{\cancel{3}}x\right) = \frac{3}{14}(60)$$
$$x = \frac{90}{7}$$

$$y = \frac{1}{3}x - 5$$
$$y = \frac{1}{3}\left(\frac{90}{7}\right) - 5$$
$$y = \frac{30}{7} - 5$$
$$y = -\frac{5}{7}$$

The solution is $\left(\frac{90}{7}, -\frac{5}{7}\right)$.

Verification

$y = \frac{1}{3}x - 5$

Left Side	Right Side
y	$\frac{1}{3}x - 5$
$-\frac{5}{7}$	$\frac{1}{3}\left(\frac{90}{7}\right) - 5$
	$\frac{30}{7} - 5$
	$-\frac{5}{7}$
LS = RS	

$x - \frac{y}{5} = 13$

Left Side	Right Side
$x - \frac{y}{5}$	13
$\frac{90}{7} - \left(\frac{-\frac{5}{7}}{5}\right)$	
$\frac{90}{7} + \frac{1}{7}$	
$\frac{91}{7}$	
13	
LS = RS	

b. $\frac{y - x}{2} = 5$ and $x + \frac{3}{4}y = 4$

$$\not{2}\left(\frac{y - x}{\not{2}}\right) = 2(5)$$
$$y - x = 10$$
$$y = 10 + x$$

$$x + \frac{3}{4}y = 4$$
$$x + \frac{3}{4}(10 + x) = 4$$
$$x + \frac{30}{4} + \frac{3}{4}x = 4$$
$$\frac{7}{4}x = 4 - \frac{30}{4}$$
$$\frac{7}{4}x = -\frac{14}{4}$$
$$\frac{\not{4}}{\not{7}}\left(\frac{\not{7}}{\not{4}}x\right) = \frac{4}{7}\left(-\frac{14}{4}\right)$$
$$x = -2$$

$$y = 10 + x$$
$$y = 10 + (-2)$$
$$y = 8$$

The solution is (–2, 8).

Verification

$\frac{y-x}{2} = 5$

Left Side	Right Side
$\frac{8-(-2)}{2}$ $\frac{8+2}{2}$ $\frac{10}{2}$ 5	5
LS = RS	

$x + \frac{3}{4}y = 4$

Left Side	Right Side
$x + \frac{3}{4}y$ $-2 + \frac{3}{4}(8)$ $-2+6$ 4	4
LS = RS	

c. $3y = \frac{1}{3} - \frac{2}{3}x$ and $x + \frac{3}{2}y = 12$

$$3(3y) = 3\left(\frac{1}{3}\right) - 3\left(\frac{2}{3}x\right)$$
$$9y = 1 - 2x$$

$$x + \frac{3}{2}y = 12$$
$$x = 12 - \frac{3}{2}y$$

$$9y = 1 - 2x$$
$$9y = 1 - 2\left(12 - \frac{3}{2}y\right)$$
$$9y = 1 - 2(12) + 2\left(\frac{3}{2}y\right)$$
$$9y = 1 - 24 + 3y$$
$$9y = -23 + 3y$$
$$6y = -23$$
$$\frac{\cancel{6}y}{\cancel{6}} = \frac{-23}{6}$$
$$y = \frac{-23}{6}$$

$$x = 12 - \frac{3}{2}y$$
$$x = 12 - \frac{3}{2}\left(\frac{-23}{6}\right)$$
$$x = 12 + \frac{23}{4}$$
$$x = \frac{71}{4}$$

The solution is $\left(\frac{71}{4}, -\frac{23}{6}\right)$.

Verification

$3y = \frac{1}{3} - \frac{2}{3}x$

Left Side	Right Side
$3y$	$\frac{1}{3} - \frac{2}{3}\left(\frac{71}{4}\right)$
$3\left(-\frac{23}{6}\right)$	$\frac{1}{3} - \frac{71}{6}$
$-\frac{23}{2}$	$-\frac{23}{2}$
LS = RS	

$x + \frac{3}{2}y = 12$

Left Side	Right Side
$x + \frac{3}{2}y$	12
$\frac{71}{4} + \frac{3}{2}\left(-\frac{23}{6}\right)$	
$\frac{71}{4} - \frac{23}{4}$	
$\frac{48}{4}$	
12	
LS = RS	

Question 6, page 475

Let x and y represent the two numbers. The system of linear equations is $x + y = 20$ and $2x = 4y + 4$.

$$x + y = 20$$
$$x = 20 - y$$

$$2x = 4y + 4$$
$$2(20 - y) = 4y + 4$$
$$40 - 2y = 4y + 4$$
$$40 = 4y + 2y + 4$$
$$40 - 4 = 6y$$
$$36 = 6y$$
$$\frac{36}{6} = \frac{\cancel{6}y}{\cancel{6}}$$
$$6 = y$$

$$x = 20 - y$$
$$x = 20 - 6$$
$$x = 14$$

The two numbers are 6 and 14.

Question 7, page 475

a. From the point of intersection on the graph, the coordinates appear to be approximately (0.3,3.7).

b. $y = 2x + 3$ and $6x + 3y = 13$

$$\begin{aligned} 6x + 3y &= 13 \\ 6x + 3(2x + 3) &= 13 \\ 6x + 6x + 9 &= 13 \\ 12x + 9 &= 13 \\ 12x &= 4 \\ \frac{\cancel{12}x}{\cancel{12}} &= \frac{4}{12} \\ x &= \frac{1}{3} \end{aligned}$$

$$\begin{aligned} y &= 2x + 3 \\ y &= 2\left(\frac{1}{3}\right) + 3 \\ y &= \frac{2}{3} + 3 \\ y &= \frac{11}{3} \text{ or } 3\frac{2}{3} \end{aligned}$$

c. The algebraic approach gives exact value answers.

Question 8, page 476

a. $0.1y = 0.3x - 1.5$ and $x - 0.2y = 5.6$

$$\begin{aligned} x - 0.2y &= 5.6 \\ x &= 5.6 + 0.2y \end{aligned}$$

$$\begin{aligned} 0.1y &= 0.3x - 1.5 \\ 0.1y &= 0.3(5.6 + 0.2y) - 1.5 \\ 0.1y &= 1.68 + 0.06y - 1.5 \\ 0.04y &= 1.68 - 1.5 \\ 0.04y &= 0.18 \\ \frac{\cancel{0.04}y}{\cancel{0.04}} &= \frac{0.18}{0.04} \\ y &= 4.5 \end{aligned}$$

$$\begin{aligned} x &= 5.6 + 0.2y \\ x &= 5.6 + 0.2(4.5) \\ x &= 6.5 \end{aligned}$$

The solution is (6.5,4.5).

b. $10(0.1y) = 10(0.3x - 1.5)$

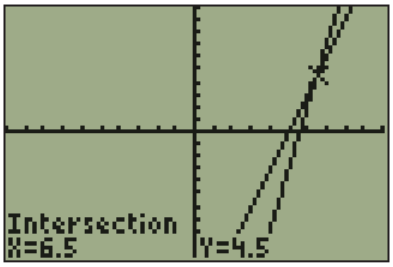
$$y = 3x - 15$$

$$\begin{aligned} 10(x - 0.2y) &= 10(5.6) \\ 10x - 2y &= 56 \\ 10x - 2(3x - 15) &= 56 \\ 10x - 6x + 30 &= 56 \\ 4x + 30 &= 56 \\ 4x &= 26 \\ \frac{\cancel{4}x}{\cancel{4}} &= \frac{26}{4} \\ x &= 6.5 \end{aligned}$$

$$\begin{aligned} y &= 3x - 15 \\ y &= 3(6.5) - 15 \\ y &= 4.5 \end{aligned}$$

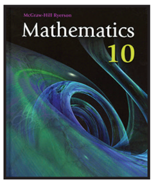
The solution is (6.5,4.5).

c.



d. Answers may vary. For example, the method in part b. eliminates the decimals and leaves numbers that are easy to work with.

Lesson 8.3: Solving Systems of Linear Equations by Elimination



Page 488, #1, 2, 3, 4, 5, and 6

Question 1, page 488

a. $x + y = 10$ and $x - y = 4$

$$\begin{array}{rrrrrr} & x & + & y & = & 10 \\ +\ (& x & - & y & = & 4\ ) \\ \hline & 2x & + & 0y & = & 14 \\ & \frac{\cancel{2}x}{\cancel{2}} & & & = & \frac{14}{2} \\ & x & & & = & 7 \end{array}$$

$$\begin{aligned} x + y &= 10 \\ 7 + y &= 10 \\ y &= 3 \end{aligned}$$

The solution is (7,3).

b. $x + 2y = 13$ and $x - y = 8$

$$x - y = 8$$
$$2(x - y) = 2(8)$$
$$2x - 2y = 16$$

$$\begin{array}{rrrrrl} & x & + & 2y & = & 13 \\ +\ (& 2x & - & 2y & = & 16\ ) \\ \hline & 3x & + & 0y & = & 29 \end{array}$$

$$\frac{\not{3}x}{\not{3}} = \frac{29}{3}$$
$$x = \frac{29}{3}$$

$$\frac{29}{3} - y = 8$$
$$-y = 8 - \frac{29}{3}$$
$$-y = -\frac{5}{3}$$
$$y = \frac{5}{3}$$

The solution is $\left(\frac{29}{3}, \frac{5}{3}\right)$.

c.
$$\begin{array}{rrrrrl} & y & - & 2x & = & -4 \\ -\ (& y & + & 3x & = & 16\ ) \\ \hline & 0y & - & 5x & = & -20 \end{array}$$

$$\frac{-\not{5}x}{-\not{5}} = \frac{-20}{-5}$$
$$x = 4$$

$$y - 2x = -4$$
$$y - 2(4) = -4$$
$$y - 8 = -4$$
$$y = 4$$

The solution is (4, 4)

Question 2, page 489

a. Rearrange $y - 3x = 11$ to $-3x + y = 11$ by changing the order of the terms on the left hand side of the first equation.

b. Rearrange $x + 7 = y$.

$$x + 7 = y$$
$$x + 7 - y = \not{y} - \not{y}$$
$$x + 7 - y = 0$$
$$x + \not{7} - \not{7} - y = 0 - 7$$
$$x - y = -7$$

c. Rearrange $4 - 3y = x$.

$$4 - 3y = x$$
$$4 - \not{3y} + \not{3y} = x + 3y$$
$$4 = x + 3y$$
$$x + 3y = 4$$

Question 3, page 489

a. The linear system is $s + a = 430$ and $10s + 13a = 4\,804$.

$$s + a = 430$$
$$-10(s + a) = -10(430)$$
$$-10s - 10a = -4\,300$$

$$\begin{array}{rrrrcr} & -10s & - & 10a & = & -4\,300 \\ +\ (& 10s & + & 13a & = & 4\,804\ ) \\ \hline & 0s & + & 3a & = & 504 \end{array}$$
$$\frac{\cancel{3}a}{\cancel{3}} = \frac{504}{3}$$
$$a = 168$$

A total of 168 tickets were sold to adults.

b. $s + a = 430$

$$s + 168 = 430$$
$$s = 262$$

A total of 262 tickets were sold to students.

Question 4, page 489

a.
$$3x + 2y = 7$$
$$4(3x + 2y) = 4(7)$$
$$12x + 8y = 28$$

$$4x + 5y = 14$$
$$3(4x + 5y) = 3(14)$$
$$12x + 15y = 42$$

$$\begin{array}{rrrrcr} & 12x & + & 8y & = & 28 \\ -\ (& 12x & + & 15y & = & 42\ ) \\ \hline & 0x & - & 7y & = & -14 \end{array}$$
$$\frac{\cancel{-7}y}{\cancel{-7}} = \frac{-14}{-7}$$
$$y = 2$$

$$3x + 2y = 7$$
$$3x + 2(2) = 7$$
$$3x + 4 = 7$$
$$3x = 3$$
$$x = 1$$

Verification

$3x + 2y = 7$

Left Side	Right Side
$3x + 2y$ $3(1) + 2(2)$ $3 + 4$ 7	7
LS = RS	

$4x + 5y = 14$

Left Side	Right Side
$4x + 5y$ $4(1) + 5(2)$ $4 + 10$ 14	14
LS = RS	

b.

$$\begin{aligned} 7x - 6y &= 27 \\ 2(7x - 6y) &= 2(27) \\ 14x - 12y &= 54 \end{aligned}$$

$$\begin{aligned} 2x + 9y &= -3 \\ 7(2x + 9y) &= 7(-3) \\ 14x + 63y &= -21 \end{aligned}$$

$$\begin{array}{rrrrrr} & 14x & - & 12y & = & 54 \\ -\,(& 14x & + & 63y & = & -21\ ) \\ \hline & 0x & - & 75y & = & 75 \\ & & & \dfrac{\cancel{-75}y}{\cancel{-75}} & = & \dfrac{75}{-75} \\ & & & y & = & -1 \end{array}$$

$$\begin{aligned} 7x - 6y &= 27 \\ 7x - 6(-1) &= 27 \\ 7x + 6 &= 27 \\ 7x &= 21 \\ x &= 3 \end{aligned}$$

Verification

$7x - 6y = 27$

Left Side	Right Side
$7x - 6y$ $7(3) - 6(-1)$ $21 + 6$ 27	27
LS = RS	

$2x + 9y = -3$

Left Side	Right Side
$2x + 9y$ $2(3) + 9(-1)$ $6 - 9$ -3	-3
LS = RS	

c.

$$\begin{aligned} 4y + 29 &= 3x \\ -3x + 4y &= -29 \\ 3(-3x + 4y) &= 3(-29) \\ -9x + 12y &= -87 \end{aligned}$$

$$\begin{aligned} 8x + 7 &= 3y \\ 8x - 3y &= -7 \\ 4(8x - 3y) &= 4(-7) \\ 32x - 12y &= -28 \end{aligned}$$

$$\begin{array}{rrrrrr} & -9x & + & 12y & = & -87 \\ +\,(& 32x & - & 12y & = & -28\ ) \\ \hline & 23x & + & 0y & = & -115 \\ & \dfrac{\cancel{23}x}{\cancel{23}} & & & = & \dfrac{-115}{23} \\ & x & & & = & -5 \end{array}$$

$$\begin{aligned} 4y + 29 &= 3x \\ 4y + 29 &= 3(-5) \\ 4y + 29 &= -15 \\ 4y &= -44 \\ y &= -11 \end{aligned}$$

Verification

$4y + 29 = 3x$

Left Side	Right Side
$4y + 29$	$3x$
$4(-11) + 29$	$3(-5)$
$-44 + 29$	-15
-15	
LS = RS	

$8x + 7 = 3y$

Left Side	Right Side
$8x + 7$	$3y$
$8(-5) + 7$	$3(-11)$
$-40 + 7$	-33
-33	
LS = RS	

Question 5, page 489

a. $3x + 2y = 10$

$$2x - y = 4$$
$$2(2x - y) = 2(4)$$
$$4x - 2y = 8$$

$$\begin{array}{rrrrrrr} & 3x & + & 2y & = & 10 & \\ +\;(& 4x & - & 2y & = & 8 &) \\ \hline & 7x & + & 0y & = & 18 & \end{array}$$

$$\frac{\cancel{7}x}{\cancel{7}} = \frac{18}{7}$$
$$x = \frac{18}{7}$$

$$2x - y = 4$$
$$2\left(\frac{18}{7}\right) - y = 4$$
$$\frac{36}{7} - y = 4$$
$$-y = 4 - \frac{36}{7}$$
$$-y = -\frac{8}{7}$$
$$y = \frac{8}{7} \text{ or } 1\frac{1}{7}$$

b. $\frac{x}{3} - y = \frac{3}{5}$ and $x + 6y = 4$

$$\frac{x}{3} - y = \frac{3}{5}$$
$$3\left(\frac{x}{3} - y\right) = 3\left(\frac{3}{5}\right)$$
$$x - 3y = \frac{9}{5}$$

$$\begin{array}{rrrrrl} & x & - & 3y & = & \frac{9}{5} \\ -(& x & + & 6y & = & 4\ ) \\ \hline & 0x & - & 9y & = & -\frac{11}{5} \end{array}$$
$$\frac{\cancel{-9}y}{\cancel{-9}} = \frac{\left(-\frac{11}{5}\right)}{-9}$$
$$y = \frac{11}{45}$$

$$x + 6y = 4$$
$$x + 6\left(\frac{11}{45}\right) = 4$$
$$x + \frac{66}{45} = 4$$
$$x + \cancel{\frac{66}{45}} - \cancel{\frac{66}{45}} = 4 - \frac{66}{45}$$
$$x = \frac{38}{15}$$

c.

$$2 - \frac{y}{2} = \frac{x}{3}$$
$$-\frac{x}{3} - \frac{y}{2} = -2$$
$$12\left(-\frac{x}{3} - \frac{y}{2}\right) = 12(-2)$$
$$-4x - 6y = -24$$

$$\frac{2}{3}(2x - 3y) = 4$$
$$\frac{4x}{3} - 2y = 4$$
$$3\left(\frac{4x}{3} - 2y\right) = 3(4)$$
$$4x - 6y = 12$$

$$\begin{array}{rrrrrl} & -4x & - & 6y & = & -24 \\ +(& 4x & - & 6y & = & 12\ ) \\ \hline & 0x & - & 12y & = & -12 \end{array}$$
$$\frac{\cancel{-12}y}{\cancel{-12}} = \frac{-12}{-12}$$
$$y = 1$$

$$4x - 6y = 12$$
$$4x - 6(1) = 12$$
$$4x - 6 = 12$$
$$4x = 18$$
$$\frac{\cancel{4}x}{\cancel{4}} = \frac{18}{4}$$
$$x = \frac{9}{2} \text{ or } 4\frac{1}{2}$$

Question 6, page 489

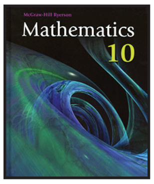
$3x + 2y = 7$ and $9x + 6y = 16$

$$\begin{aligned} 3x + 2y &= 7 \\ 3(3x + 2y) &= 3(7) \\ 9x + 6y &= 21 \end{aligned}$$

$$\begin{array}{rrrrrrr} & 9x & + & 6y & = & 21 & \\ -\ (& 9x & + & 6y & = & 16 &) \\ \hline & 0x & - & 0y & = & -5 & \\ & & & 0 & = & -5 & \end{array}$$

The result of $0 = -5$ is a false statement, so the system has no solution. A graph of these equations would result in two parallel lines, and thus no point of intersection.

Lesson 8.4: Solving Problems using Systems of Linear Equations



Page 440, #1, 2, 3, 5, and 10

Page 498, #1, 6 , 7, 8, 9, and 11

Question 1, page 440

a. Let C represent the cost, in dollars. Let s represent the number of songs downloaded. The system can be modelled by the equations $C = 0.99s$ and $C = 0.79s + 11$.

b. Let h represent the height above ground, in metres. Let t represent the time, in minutes. The system can be modelled by the equations $h = 800 - 55t$ and $h = 80t$.

c. Let R represent the amount of material sorted, in tonnes. Let t represent the time, in hours. The system can be modelled by the equations $R = 100 + 20t$ and $R = 40t$.

Question 2, page 440

a. Let J represent Jamal's age, in years. Let M represent Maria's age, in years. The system can be modelled by the equations $J = 3M$ and $J + 7 = 2(M + 7)$.

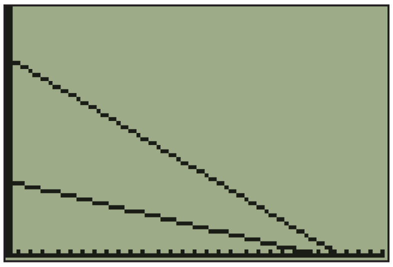
b. Let C represent the temperature, in degrees Celsius. Let r represent the time, in hours. The system can be modelled by the equations $C = 2 - 2t$ and $C = -8 + 4t$.

Question 3, page 440

a. Let G represent the number of goals Molly has scored. Let A represent the number of assists Molly has earned. The system can be modelled by the equations $G + A = 32$ and $3G = A$.

Question 5, page 441

a. Let V represent the volume of water remaining in each tank, in litres. Let t represent the time, in minutes. The system can be modelled by the equations $V = 800 - 30t$ and $V = 300 - 12t$.

b. 

c.

$V = 800 - 30t$	$V = 300 - 12t$
$0 = 800 - 30t$	$0 = 300 - 12t$
$30t = 800$	$12t = 300$
$t = \frac{800}{30}$	$t = \frac{300}{12}$
$t = 26\frac{2}{3}$	$t = 25$

The larger tank starts with more water in it and empties in 26 min 40 s, while the smaller tank drains more slowly, but starts with less water in it. The smaller tank empties in 25 min. The two graphs do not intersect until after 27 min (below the x-axis), so they never contain the same amount of water before they are both empty.

Question 10, page 442

Let t represent the time, in hours and let d represents the distance from Hall Beach, in kilometres.

The system can be modelled by equations $d = 15 + 25t$ and $d = 35 - 40t$.

When Mary and Amaruk pass each other, they will be about 22.7 km from Hall Beach.

Question 1, page 498

a. $2x - 5y = 12$ and $-7x + 5y = 48$

Elimination is best for this system because the equations can be added immediately to eliminate y.

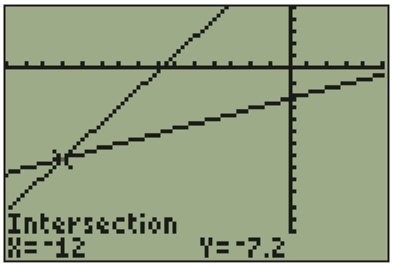
$$\begin{array}{rrcrcr} & 2x & - & 5y & = & 12 \\ + (& -7x & + & 5y & = & 48) \\ \hline & -5x & + & 0y & = & 60 \end{array}$$

$$\frac{\cancel{-5}x}{\cancel{-5}} = \frac{60}{-5}$$

$$x = -12$$

$$\begin{aligned} 2x - 5y &= 12 \\ 2(-12) - 5y &= 12 \\ -24 - 5y &= 12 \\ -5y &= 36 \\ \frac{\cancel{-5}y}{\cancel{-5}} &= \frac{36}{-5} \\ y &= -7.2 \end{aligned}$$

Verification by graphing.



b. $3y = 6 - x$ and $5x + 6y = -6$

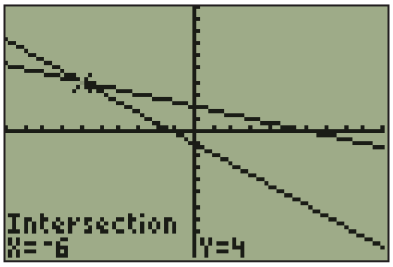
Substitution is best for this system because x can be easily isolated in the first equation.

$$\begin{aligned} 3y &= 6 - x \\ 3y - 6 &= -x \\ -3y + 6 &= x \end{aligned}$$

$$\begin{aligned} 5x + 6y &= -6 \\ 5(-3y + 6) + 6y &= -6 \\ -15y + 30 + 6y &= -6 \\ -9y + 30 &= -6 \\ -9y &= -36 \\ \frac{\cancel{-9}y}{\cancel{-9}} &= \frac{-36}{-9} \\ y &= 4 \end{aligned}$$

$$\begin{aligned} -3y + 6 &= x \\ -3(4) + 6 &= x \\ -12 + 6 &= x \\ -6 &= x \end{aligned}$$

Verification by graphing.



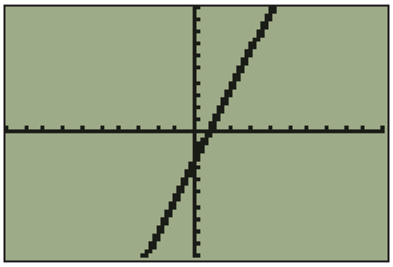
c. $n = 3k - 2$ and $2n - 6k = -4$

Substitution is best for this system because $3k - 2$ can be substituted immediately for n in the second equation.

$$\begin{aligned} n &= 3k - 2 \\ 2n - 6k &= -4 \\ 2(3k - 2) - 6k &= -4 \\ 6k - 4 - 6k &= -4 \\ -4 &= -4 \end{aligned}$$

Since both variables were eliminated, and a true statement results, the solution is the infinite number of values that satisfy the equation $n = 3k - 2$. When graphed, the two lines will be coincident.

Verification by graphing.



Question 6, page 499

$C = 0.75 + 0.0072h$ and $C = 4 + 0.0018h$

$$0.75 + 0.0072h = 4 + 0.0018h$$
$$0.0072h = 4 + 0.0018h - 0.75$$
$$0.0072h - 0.0018h = 3.25$$
$$0.0054h = 3.25$$
$$\frac{0.0054h}{0.0054} = \frac{3.25}{0.0054}$$
$$h = 601.8518...$$

It will take approximately 602 hours for the compact fluorescent bulb to be less expensive than the incandescent bulb.

Question 7, page 499

For premium seating at the circus, let a represent the number of adult tickets sold and let s represent the number of student tickets sold.

The equation $a + s = 130$ represents the total number of premium seats sold. The equation $250a + 175s = 29\,125$ represents the total revenue, in dollars, from the sale of the premium seats.

$$a + s = 130$$
$$a = 130 - s$$

$$250a + 175s = 29\,125$$
$$250(130 - s) + 175s = 29\,125$$
$$32\,500 - 250s + 175s = 29\,125$$
$$32\,500 - 75s = 29\,125$$
$$-75s = 29\,125 - 32\,500$$
$$-75s = -3\,375$$
$$\frac{-75s}{-75} = \frac{-3\,375}{-75}$$
$$s = 45$$

$$a = 130 - s$$
$$a = 130 - 45$$
$$a = 85$$

In the premium seating at the circus, there were 85 adults and 45 students.

Question 8, page 499

Let C represents the cost in dollars, for renting a car and let n be the distance driven, in kilometres. The equation $C = 379 + 0.1n$ represents the cost of renting from Speed-E-Car rental. The equation $C = 249 + 0.35n$ represents the cost of renting from Easy 4 U rental.

$$379 + 0.1n = 249 + 0.35n$$
$$0.1n - 0.35n = 249 - 379$$
$$-0.25n = -130$$
$$\frac{-0.25n}{-0.25} = \frac{-130}{-0.25}$$
$$n = 520$$

At 520 km, the cost of renting from each company is equal. Since Speed-E-Car Rental charges less per kilometre, Speed-E-Car Rental will become less expensive after 520 km, and will be more expensive if less than 520 km are driven.

Question 9, page 500

a. The population of fish in the lake is decreasing by 1 000 each year, while the number of fish eaten by osprey is increasing by 200 each year.

b. Let F represents the number of fish and let x be the year. Both linear functions have an equation of the form $F = mx + b$.

For the fish in the lake, using the points (1,10 000) and (2,9 000) gives the following system:

$$10\,000 = m(1) + b$$
$$9\,000 = m(2) + b$$

$$\begin{array}{rrcrcrl} & 10\,000 & = & m & + & b & \\ -\ (& 9\,000 & = & 2m & + & b &) \\ \hline & 1000 & = & -m & + & 0b & \\ & 1000 & = & -m & & & \\ & -1000 & = & m & & & \end{array}$$

$10\,000 = m(1) + b$

$10\,000 = -1\,000(1) + b$

$10\,000 = -1\,000 + b$

$11\,000 = b$

Substitute the values of m and b into the equation $F = mx + b$.

$F = -1\,000x + 11\,000$

Therefore, $F = -1\,000x + 11\,000$ represents the number of fish in the lake.

For the fish eaten by osprey, using the points (1,700) and (2,900) gives the following system:

$700 = m(1) + b$

$900 = m(2) + b$

$$\begin{array}{rrcrcrl} & 700 & = & m & + & b & \\ -\,(& 900 & = & 2m & + & b &) \\ \hline & -200 & = & -m & + & 0b & \\ & 200 & = & m & & & \end{array}$$

$700 = m(1) + b$

$700 = 200(1) + b$

$700 = 200 + b$

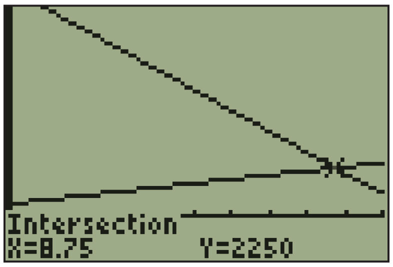
$500 = b$

Substitute the values of m and b into the equation $F = mx + b$.

$F = 200x + 500$

Therefore, $F = 200x + 500$ represents the number of fish eaten by osprey.

c. The intersection point indicates that after 8.75 years, the number of fish in the lake will equal the number of fish that are eaten by osprey.



d. As the number of fish decreases, the osprey population will also decrease because there will not be enough fish to keep feeding the osprey.

Question 11, page 501

Let c represents the amount of time, in minutes, that Juan spent cross-country skiing.

Let s represent the amount of time, in minutes, that Juan spent playing squash. The system of equations representing this situation is as follows:

$c + s = 100$ and $50c + 42s = 4\,850$

$$c + s = 100$$
$$50(c + s) = 50(100)$$
$$50c + 50s = 5\,000$$

$$\begin{array}{rrrrr} & 50c & + & 50s & = & 5\,000 \\ -\,(& 50c & + & 42s & = & 4\,850\ ) \\ \hline & 0c & + & 8s & = & 150 \end{array}$$

$$8s = 150$$
$$\frac{\not{8}s}{\not{8}} = \frac{150}{8}$$
$$s = 18.75$$

$$c + s = 100$$
$$c + 18.75 = 100$$
$$c = 100 - 18.75$$
$$c = 81.25$$

Juan cross-country skied for 81.25 min and played squash for 18.75 min.